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MAXIMUM POWER TRANSFER IN NON-LINEAR SOURCE–LOAD SYSTEMS

G. J. VACHTSEVANOS AND K. C. KALAITZAKIS

Department of Electrical Engineering, School of Engineering, Democritus University of Thrace, Xanthi, Greece

SUMMARY

This paper is concerned with the conceptual design and realization of matching networks for the continuous transfer of maximum power from a non-linear source with randomly varying characteristics to a load. Such sources are commonly encountered in the use of photovoltaic arrays or wind energy conversion systems for the production of electric power. Experimental studies as well as computer simulation results verify the validity of the design and point to methods for its practical implementation.

INTRODUCTION

In recent years, increased interest in the use of wind, solar and other renewable energy sources for electricity production has resulted in a re-examination of the problem of transferring maximum power from a non-linear source whose characteristics are functions of some randomly varying external parameters to a load with a fixed configuration. Typical examples of this situation are found in connection with wind electric conversion systems or photovoltaic (PV) generators interconnected with the utility grid.[1–3] In the first case, wind speed is the external parameter influencing the generator output characteristics whereas in the second, the current–voltage relation of the photovoltaic array is dependent upon the insolation level. In both cases, the grid behaviour at the point of interconnection is assumed to be fairly constant, and maximum electrical power is to be transferred from the generator to the utility network for any wind speed or insolation values.

Several approaches to the power matching problem have been proposed. One method involves the manipulation of the source characteristics, if possible, through a set of control variables so that source–load matching is achieved.[4,5] A second approach incorporates a suitable matching device between source and load which, through appropriate control action, accomplishes maximum power transfer.[6,7]

This paper is concerned with a particular realization of this second scheme and its application to renewable energy sources such as wind energy conversion devices or photovoltaic array systems.

PROBLEM DEFINITION

Figure 1(a) shows typical voltage–current characteristics for several sources of practical importance. When any one of these sources is directly connected to a load, R_L, it is obvious that at some operating condition specified by the value of the load resistance R_L and the source characteristics, the power $P_L = vi$ transferred from the source to the load is maximized. Figure 1(b) demonstrates the dependence of the v–i characteristics of a photovoltaic source on the intensity of insolation. With an insolation level H_1, a value of the load resistor equal to R_L ensures maximum power transfer. An increase of the insolation level to H_2 or H_3 results in a new operating point which does not necessarily guarantee maximum power transfer.

A possible solution to the problem of continuously matching source–load characteristics involves the design of an appropriate device which, when inserted between generator and load, maximizes the power absorbed by the latter, regardless of the value of any external parameter influencing the source behaviour. As additional design constraints, simplicity and low cost must characterize this matching device.

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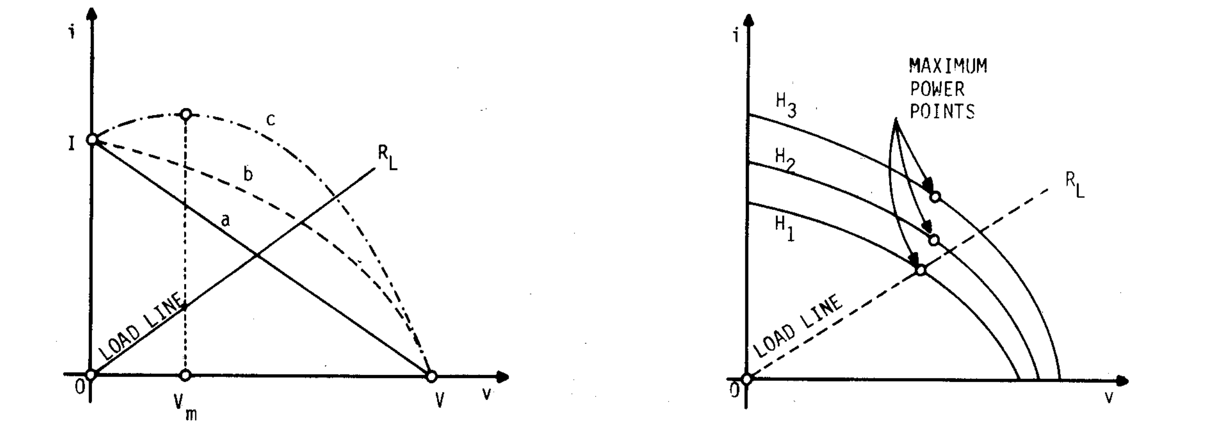

Figure 1(a). Voltage–current characteristics of: a. A linear source. b. A photovoltaic array. c. A wind generator

Figure 1(b). Dependence of v–i characteristics of a photovoltaic source on the intensity of solar irradiation (H_1, H_2, H_3)

UNIQUENESS OF MAXIMUM POWER POINT

It is, of course, well known that the maximum power point for a linear source (curve a in Figure 1(a)) is unique and corresponds to a load resistance value equal to the internal resistance of the source.

Curve b in Figure 1(a) represents the v–i characteristics of a PV array at some fixed insolation level. The functional relationship $i=i(v)$, depicted in this Figure, must obey the following constraints for a physically realizable source:

$$
\begin{aligned}
& i(v) \geqslant 0 \quad \forall v \in [0, V] \\
& \frac{\mathrm{d}i(v)}{\mathrm{d}v} \leqslant 0 \quad \forall v \in [0, V] \\
& \frac{\mathrm{d}^2 i(v)}{\mathrm{d}v^2} < 0 \quad \forall v \in [0, V] \\
& i(0) = I \quad \text{and} \quad i(V) = 0
\end{aligned}
\tag{1}
$$

If $P(v)=vi$ is the photovoltaic power output and since $P(0)=P(V)=0$, according to Rolle's theorem, $P(v)$ has at least one extremum in the interval $[0, V]$. Therefore

$$
P(v) \geqslant 0 \quad \forall v \in [0, V] \tag{2}
$$

and the points $v=0$ and $v=V$ correspond to minimum power in the interval $[0, V]$. Moreover, the derivative of $P(v)$ is given by

$$
\frac{\mathrm{d}P(v)}{\mathrm{d}v} = i(v) + v\frac{\mathrm{d}i(v)}{\mathrm{d}v} \tag{3}
$$

and the second derivative is

$$
\frac{\mathrm{d}^2P(v)}{\mathrm{d}v^2} = 2\frac{\mathrm{d}i(v)}{\mathrm{d}v} + v\frac{\mathrm{d}^2 i(v)}{\mathrm{d}v^2} \tag{4}
$$

Equation (4), in conjunction with constraints (1), takes on negative values in the interval $[0, V]$ and as a consequence all roots of the first derivative of $P(v)$ are maxima of $P(v)$. This implies that there is a unique maximum for the power transferred to the load in the interval $[0, V]$. Any change in insolation values results in shifting the characteristic curve upward or downward without changing its shape. For every given value of insolation, therefore, there is a unique maximum power point associated with the photovoltaic array output.

Curve c of Figure 1(a) shows a typical wind generator output characteristic for a given wind velocity. From physical considerations again, the function $i(v)$ and its derivatives must obey the following set of constraints:

$$
\begin{aligned}
&i(v) \geqslant 0 \quad \frac{\mathrm{d}i(v)}{\mathrm{d}v} > 0 \quad \forall v \in [0, V_{\mathrm{m}}) \\
&i(v) \geqslant 0 \quad \frac{\mathrm{d}i(v)}{\mathrm{d}v} < 0 \quad \forall v \in (V_{\mathrm{m}}, V] \\
&\frac{\mathrm{d}i(v)}{\mathrm{d}v} = 0 \quad v = V_{\mathrm{m}} \\
&\frac{\mathrm{d}^2 i(v)}{\mathrm{d}v^2} < 0 \quad \forall v \in [0, V]
\end{aligned}
\tag{5}
$$

It is true, therefore, that $P(v) = vi = 0$ for $v = 0$ or $v = V$ and the function $P(v)$, according to Rolle's theorem, must have at least one extremum in the interval $[0, V]$. The first and second derivatives of $P(v)$ are given by

$$\frac{\mathrm{d}P(v)}{\mathrm{d}v} = i(v) + v\frac{\mathrm{d}i(v)}{\mathrm{d}v} \tag{6}$$

$$\frac{\mathrm{d}^2 P(v)}{\mathrm{d}v^2} = 2\frac{\mathrm{d}i(v)}{\mathrm{d}v} + v\frac{\mathrm{d}^2 i(v)}{\mathrm{d}v^2} \tag{7}$$

and since $i(v) > 0$ and $\mathrm{d}i(v)/\mathrm{d}v \geqslant 0$ in $[0, V_{\mathrm{m}}]$, it must be true that

$$\frac{\mathrm{d}P(v)}{\mathrm{d}v} > 0 \quad \forall v \in [0, V_{\mathrm{m}}] \tag{8}$$

which implies that there is no root of the derivative of the function $P(v)$ in this same interval. In the interval $[V_{\mathrm{m}}, V]$, the second derivative of $P(v)$ must satisfy the relation

$$\frac{\mathrm{d}^2 P(v)}{\mathrm{d}v^2} < 0 \quad \forall v \in [V_{\mathrm{m}}, V] \tag{9}$$

according to the constraint set (5). Consequently, the roots of the first derivative of $P(v)$ are maxima of $P(v)$ in the interval $[V_{\mathrm{m}}, V]$ and accordingly there is a unique maximum in this interval.

A similar argument may be presented for other devices with non-linear v–i characteristics, such as a fuel cell, which are used as electric energy sources.

THE MATCHING CIRCUIT

Two generic forms for the implementation of the maximum power transfer matching network are proposed. The first is shown in Figure 2(a) and is valid when the internal resistance of the source is greater than the load resistance, whereas the second, shown in Figure 2(b), is used for the complementary case. Reference will be made below only to the first design form; analysis of the complementary case proceeds along similar steps.

The design philosophy is based upon a dynamic matching of the source–load characteristics under varying parametric conditions. This matching may be achieved by manipulating such control variables as the value of the capacitance (inductance), the frequency of operation of the electronic switch s and its duty cycle. The load is assumed to be a reactive impedance although this assumption does not imply any loss of generality.

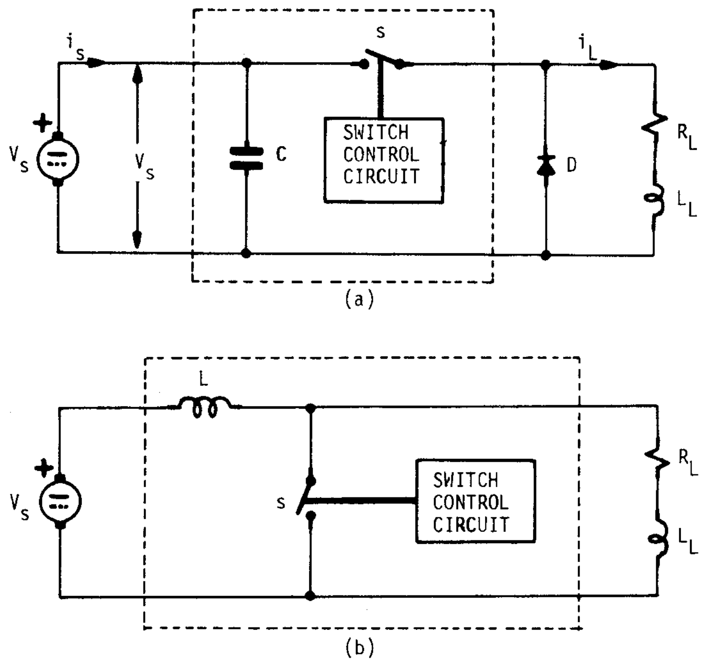

Figure 2. Implementation of maximum power transfer matching circuit

If the source current i_s is related to the source voltage v_s via a relationship of the form $i_s = f(v_s)$, then the circuit of Figure 2(a) is described by the following state equations:

(a) with the switch s open:

$$\frac{\mathrm{d}v_s}{\mathrm{d}t} = \frac{f(v_s)}{C}$$
$$\frac{\mathrm{d}i_L}{\mathrm{d}t} = -\frac{R_L}{L_L} i_L \tag{10}$$

(b) with the switch s closed:

$$\frac{\mathrm{d}v_s}{\mathrm{d}t} = \frac{f(v_s) - i_L}{C}$$
$$\frac{\mathrm{d}i_L}{\mathrm{d}t} = \frac{v_s - i_L R_L}{L_L} \tag{11}$$

A fourth-order Runge–Kutta routine is used for the solution of each set of equations (10) and (11). The solution of equations (10) is used as the initial condition for the second set (11); the subsequent solution of (11) constitutes a new initial state for (10) over the next solution interval, and so on. At each time step, the energy dissipated in the load resistance R_L is calculated. The capacitance C, the switching frequency f and the duty cycle DC are the circuit design parameters. An optimization algorithm is employed and values for the parameter set $[C, f, \mathrm{DC}]$ are computed so that the average power dissipated in the load resistance R_L is maximized.

Of practical significance is the case where the source is connected to the utility grid. For this reason the equivalent problem is stated below and the implementation procedure together with results are detailed. Figure 3 shows a network configuration for transferring maximum power from a d.c. source to the electric grid. The correspondence of this configuration to the more general case treated in previous

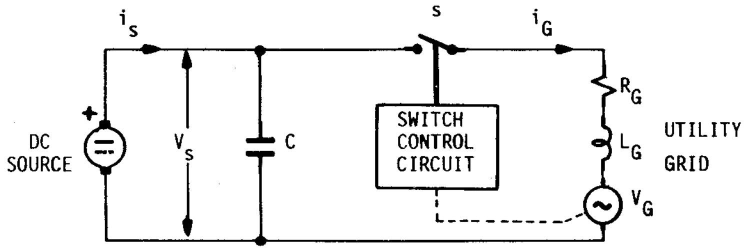

Figure 3. Implementation of matching circuit between the d.c. source and the utility grid

paragraphs is obvious; it is chosen here as a specific example because of its practical implications. The d.c. source may be a photovoltaic array, a suitable wind energy conversion device or any other generator with output characteristics similar to those shown in Figure 1(a) (i.e. fuel cells or batteries). The state equations for this system are:

(a) with the switch s open:

$$\frac{dv_s}{dt}=\frac{f(v_s)}{C}$$
$$\frac{di_G}{dt}=0 \quad \text{and} \quad i_G=0 \tag{12}$$

(b) with the switch s closed:

$$\frac{dv_s}{dt}=\frac{f(v_s)-i_G}{C}$$
$$\frac{di_G}{dt}=\frac{v_s-v_G-i_G R_G}{L_G} \tag{13}$$

where R_G and L_G are the ohmic resistance and the self inductance corresponding to the grid impedance Z_G, respectively. In the case where a series inductor is used to shape the current waveform, L_G represents the total branch inductance. Similarly, R_G includes the ohmic resistance of connectors, cables and the coil if present.

Since the switch s is in reality a d.c. to a.c. inverter consisting of a bridge of thyristors, the firing of the thyristor gates must be synchronized with the grid voltage waveform. A suitable control voltage V_c regulates the time period T_d between the zero crossing of the grid voltage and the switch closing (firing of the appropriate thyristor pair).

A similar simulation methodology to the one described above for the general case is used for the detailed study of this system. A flow chart of the algorithmic steps is shown in Figure 4. Input data relate to the impedance of the power network Z_G, the self inductance of the corrective inductor L, the capacitance C and some arbitrary initial setting of the control voltage V_c. The quantities R_G, L_G and an initial state are computed first. Next, a Runge–Kutta routine is used repeatedly to solve the state equations and bring the system to a steady-state condition. This part of the program is provided with information about the nature of the system state equations and the d.c. source from appropriate routines. The d.c. source routine receives input data for such external parameters as wind speed, insolation level, ambient temperature, etc. The results of the Runge–Kutta routine are used to calculate the values of the input power to the inverter (switch) P_{in}, the system output power P_{out}, the r.m.s. values of the output voltage and current V_{rms} and I_{rms}, respectively, the apparent power S, the inverter efficiency Eff and the power factor PF. The program then checks to verify if the output power P_{out} has reached a relative maximum level. In the event that this is not true, an optimization loop determines a new value for the control signal V_c which maximizes P_{out}.

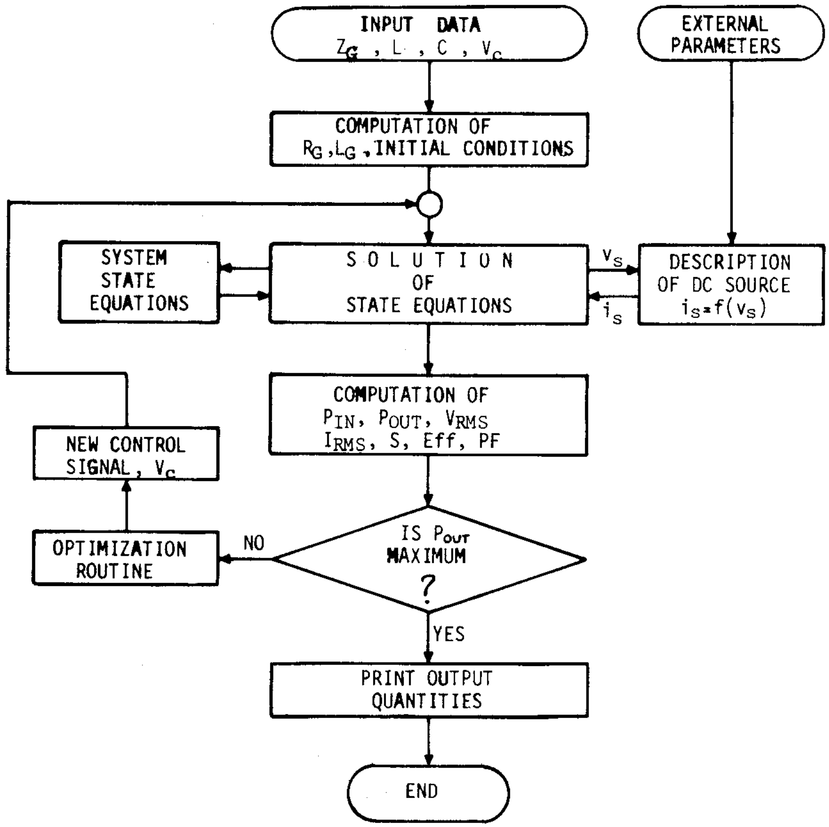

Figure 4. Flow chart of the computer simulation model described in Figure 3

IMPLEMENTATION AND RESULTS

Figure 5 is an expanded version of Figure 3 showing in more detail the nature and function of the matching network. The d.c. source may be any one of the four alternative devices depicted in the Figure. The line-commutated inverter consists basically of a conventional thyristor bridge. The conduction angle of each conducting pair of thyristors (and consequently the duty cycle) is controlled via a feedback loop which samples a measure of the output power and generates an appropriate control signal. The latter initiates firing of the thyristors so that maximum power is always transferred from the source to the load. That is the matching network here performs a dual function: it converts the d.c. output of the source to a.c. form while, at the same time, it accomplishes the task of transferring maximum power from the generator to the utility lines. In this particular implementation, inverter action relies upon synchronization of the switching frequency f with the power system frequency (line-commutated inverter). Furthermore, practical considerations and a reduced parametric sensitivity, as shown below, dictate fixed values for the capacitance C and the inductance L. Thus, the only design parameter free to be manipulated in the optimization algorithm is the duty cycle DC.

The scheme described above has been realized in the laboratory with a photovoltaic simulator substituting for the d.c. source.[8] A parallel computer simulation of the entire system operation was used to allow for required flexibility in sensitivity analyses and system configuration options. The approach followed emphasizes the dual role of analysis and experimentation. Simulation results were verified with experimental data and the design of specific experiments was guided by analytical studies. Figure 6 shows typical computer results. Figure 6(a) depicts the relationship between the output power P_{out} and the insolation level, H for two different operating conditions: (a) without the maximum power tracking network, and

(b) with the maximum power tracker (MPT) in operation. The parameters network impedance, inductance and capacitance are considered to be constant. After a threshold level of insolation, the improvement in power output with the MPT in place is as high as 50 per cent for the system considered. Experimental results verify these model predictions.

Various parametric sensitivity runs are shown in Figure 6(b). Inverter efficiency (Eff) and the power factor (PF) at the point of interconnection are plotted as functions of the self inductance L of the coil and the capacitance C. In all cases, Eff and PF are compared with and without the matching network in operation. It is observed that, generally, introduction of the MPT results in improved efficiency and power factor levels. This improvement is more pronounced for smaller values of L and C. The grid impedance at the point of interconnection is taken to be constant at a value of $Z_G = 0.005\ \angle 60°\ \Omega$. The values for the parameters L and C, when kept alternatively constant, are fixed at 1,000 μH and 1,000 μF, respectively. For low values of L and C, system sensitivity is relatively high and decreases substantially when these two parameters reach levels of 3,000 μH and 3,000 μF, respectively.

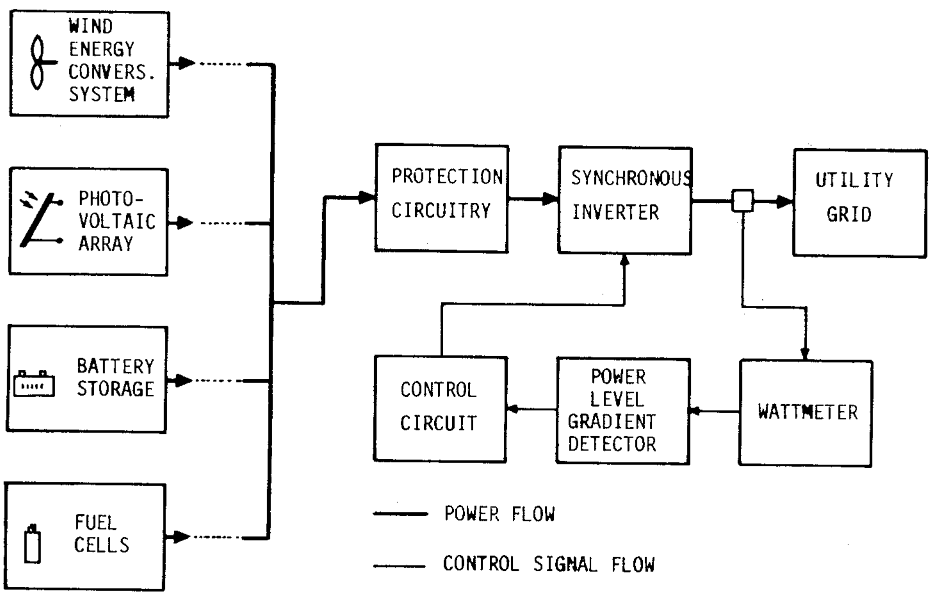

Figure 5. Block diagram of the complete system under consideration. Matching feedback loop is shown in more detail

For any given application therefore, a proper choice for the values of L and C will ensure optimum operation; that is maximization of power transferred from the source to the load and minimization of the total harmonic distortion injected into the load by the power conditioning unit. For maximum power transfer only, the optimum value of L is zero. A compromise design for the specific application referred to above, under typical solar radiation conditions, leads to a value for L equal to 1,000 μH. Any value for the capacitance C larger than 3,000 μF will guarantee maximum power transfer with minimum sensitivity to solar radiation changes. In practical applications, values in the range of 3,000–15,000 μF have been successfully used.

Similar improvement in performance characteristics has been achieved with a 2.5 kW wind energy conversion system substituting for the d.c. source. The output of a variable speed a.c. generator is rectified and subsequently converted to a.c. form via a synchronous inverter. Analytical studies support the conclusions reached for all cases considered.

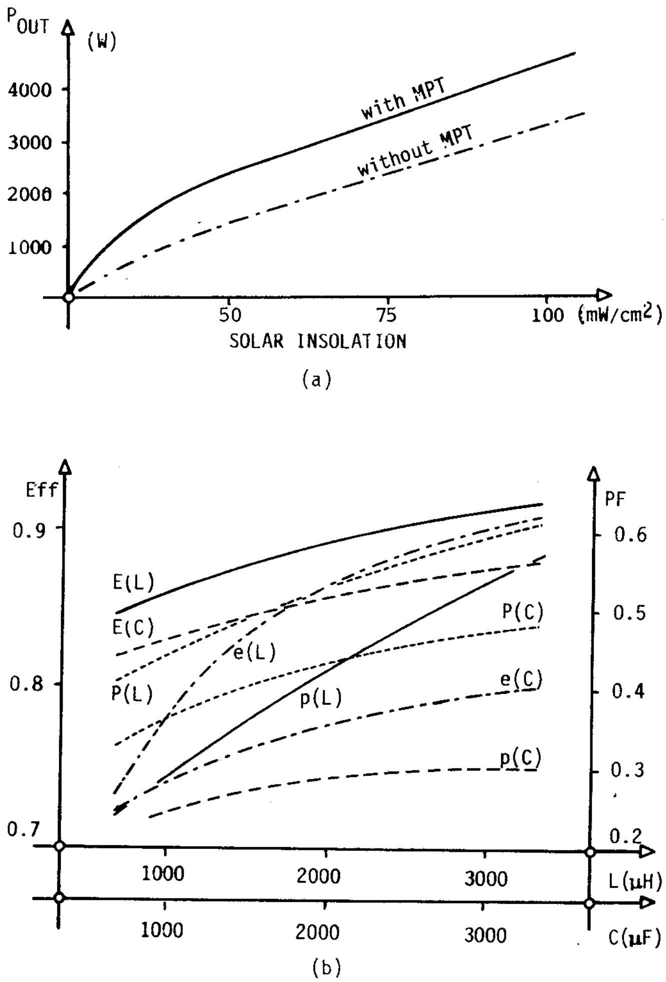

Figure 6. Results for the simulated model of the system interconnecting a photovoltaic array to the utility grid. $E(L)\{e(L)\}$ = efficiency versus coil inductance with {without} MPT; $E(C)\{e(C)\}$ = efficiency versus capacitance with {without} MPT; $P(L)\{p(L)\}$ = power factor versus coil inductance with {without} MPT; $P(C)\{p(C)\}$ = power factor versus capacitance with {without} MPT. $Z_G = 0{\cdot}005\ \Omega/60$; $L = 1{,}000\ \mu H$; $C = 2{,}200\ \mu F$

CONCLUSIONS

A maximum power transfer scheme for non-linear source–load configurations is presented. It is shown, via theoretical considerations, that for such practical sources as photovoltaic arrays or wind energy conversion systems the power characteristics curve has a unique maximum. This maximum may be achieved via a matching network whose design parameters are manipulated so that maximum power is transferred to the load. Experimental results and simulation studies indicate that the practical realization of the matching network conceptual design performs according to predicted standards. The paper focuses upon the link between theoretical and practical considerations associated with the design of networks for maximum power transfer from a non-linear source with varying parameters to a load.

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