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OPTIMAL PV SYSTEM DIMENSIONING WITH OBSTRUCTED SOLAR RADIATION

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Abstract—This paper describes an analytical methodology for the optimised design of stand-alone photovoltaic (PV) systems installed at locations where the solar radiation is considerably shortened by obstacles, i.e. at the bottom of a gorge. A method for the computation of the real available solar energy incident on the PV panel is proposed, based on easily conducted measurements. The optimum tilt of the PV array, under such conditions, is obtained and a dimensioning procedure provides the optimal size of the PV-array/storage. The resulting system is tested by a verification routine. A case study, employing the complete algorithm proposed, is illustrated.

1. INTRODUCTION

In recent years, an increasing number of electricity consumers are powered by photovoltaic (PV) devices, when the utility grid is unavailable. The relatively high cost of both the PV array and the battery for reliable operation necessitates the development of an accurate method for the estimation of their size, minimizing the investment cost of the installation. There are many approaches to the solution of the dimensioning problem for flat terrains [1–5]. Some investigators have proposed methods for load matching and management, whenever possible, to reduce the PV size [6, 7].

In many cases, the solar radiation incident on the PV surface is considerably reduced by obstacles that cannot be avoided. One such application is the installation of a PV array on the bottom of a gorge. A method for the computation of the real available solar radiation on the PV panel is proposed here.

The tilt of the PV surface to the horizontal plane plays an important role in the amount of solar energy captured by the system. A consistent method for the computation of one or two optimal slopes for the PV surface is reported here [8].

A simplified method is proposed for the computation of the optimal dimensions of the system. Some solar radiation data and the PV system specifications are required as inputs. The outputs are the optimal PV array rated power and the battery capacity required. A verification procedure is proposed for testing the sufficiency of the resulting PV system.

Finally, an application example is illustrated for a PV system located at the bottom of the Samaria gorge on the island of Crete, Greece, supplying an observatory.

2. SUNRISE AND SUNSET TIME ALTERATION BY OBSTACLES

Solar radiation is blocked by obstacles in some applications, considerably reducing the total energy incident on the PV array in a 24-h period. A method for the calculation of the shifted sunrise and sunset times is proposed here (Fig. 1).

Let h be the height of the obstacle and γ_f be the angle between its direction and the north–south axis, as shown in Fig. 1. Considering the right-angled triangles in the figure, with edges (h, a) and (d, a), formed by the first solar beam, after sunrise, reaching point A, the following relations can be written:

$$d = a \cdot \sin(\gamma_f - \gamma_s)$$

and

$$h = a / \tan \theta_z, \qquad (1)$$

where d is the distance from the obstacle, θ_z is the zenith angle and γ_s is the solar azimuth angle, given by:

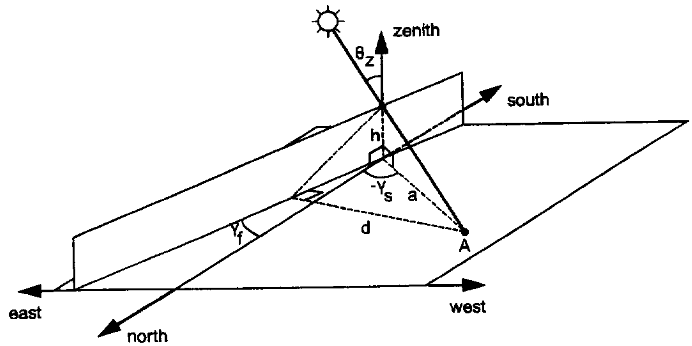

Fig. 1. Sunrise hour angle change by an obstacle.

$$\cos\theta_z = \sin\phi\sin\delta + \cos\phi\cos\delta\cos\omega$$

$$\sin\gamma_s = \frac{\cos\delta\sin\omega}{\sin\theta_z}$$

and

$$\omega = 15^\circ(t-12), \tag{2}$$

where ω is the solar hour angle, ϕ is the latitude, δ is the declination and t is the time of the day. Dividing relations (1) and replacing r by d/h, yields:

$$r \equiv \frac{d}{h} = \sin(\gamma_f - \gamma_s)\tan\theta_z. \tag{3}$$

If the ratio r and the angle γ_f of the obstacle are known, the sunrise time t_{sr}, for any day, can be calculated solving the set of eqs (2) and (3) for t. The time t in all relations in this work is expressed as local apparent time (LAT). If the characteristics of the obstacles (r and γ_f) are not known or if they are difficult to measure directly, an alternative process is proposed. In this case, two observations of the sunrise time (at point A) on two different days of the year (t_{sr1}, t_{sr2}) are required. Then the two pairs (θ_{z1}, γ_{s1}) and (θ_{z2}, γ_{s2}) can be computed with eq. (2), and eq. (3) giving:

$$\sin(\gamma_f - \gamma_{s1})\tan\theta_{z1} - \sin(\gamma_f - \gamma_{s2})\tan\theta_{z2} = 0. \tag{4}$$

Equation (4) can be solved for γ_f by any appropriate numerical method (i.e. secant root finding method). Using this value of γ_f the value of r can be calculated by eq. (3). The same procedure can be performed if an obstacle affects the sunset time. If this is the case, the term $(\gamma_f - \gamma_s)$ must be replaced by $(\gamma_s - \gamma_f)$ in all the above relations.

3. SOLAR RADIATION BETWEEN OBSTACLES

The total solar irradiance during one day, $G(t)$ [kW/m^2], from sunrise (t_{sr}) to sunset (t_{ss}) is described by the curve of Fig. 2. The relation between the irradiance and time t is:

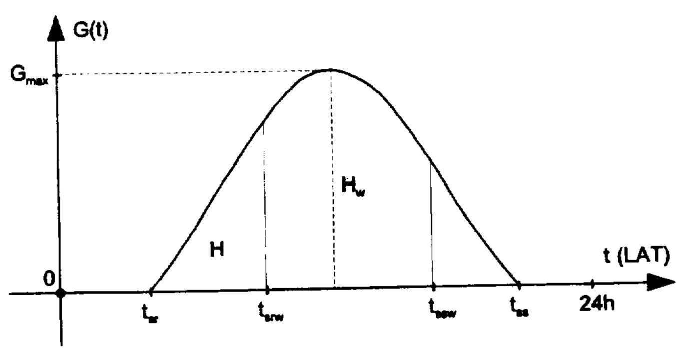

Fig. 2. Daily solar radiation between obstacles.

$$G(t) = M\left[b\cos^2\left(\frac{\pi t}{12}\right) - (a - b\cos\omega_{sr})\cos\left(\frac{\pi t}{12}\right) - a\cos\omega_{sr}\right] \quad (5)$$

and

$$M = \frac{\pi H_n}{24\left(\sin\omega_{sr} - \frac{\pi\omega_s}{180}\cos\omega_{sr}\right)}$$

$$a = 0.409 + 0.5016\sin(\omega_{sr} - 60°)$$

$$b = 0.6609 - 0.4767\sin(\omega_{sr} - 60°), \quad (6)$$

where H_n [kWh/m^2] is the measured total solar radiation during the nth day of the year. The result of integration of eq. (5) is the solar energy in the integration interval. The portion between the vertical lines of t_{srw} and t_{ssw} in Fig. 2 represents the available daily solar energy $H_{w,n}$ if some obstacles shift the sunrise and sunset times to t_{srw} and t_{ssw}, respectively.

The ratio of the daily solar radiation with and without obstacles is found to be:

$$\rho = \frac{H_{w,n}}{H_n} = \frac{\pi}{24\left(\sin\omega_{sr} - \frac{\pi\omega_{sr}}{180}\cos\omega_{sr}\right)} \times\left[\left(\frac{b}{2} - a\cos\omega_{sr}\right)(t_{ssw} - t_{srw}) - \frac{12(a - b\cos\omega_{sr})}{\pi}\left(\sin\left(\frac{\pi t_{ssw}}{12}\right) - \sin\left(\frac{\pi t_{srw}}{12}\right)\right) + \frac{3b}{\pi}\left(\sin\left(\frac{\pi t_{ssw}}{6}\right) - \sin\left(\frac{\pi t_{srw}}{6}\right)\right)\right]. \quad (7)$$

The ratio ρ is a function of the known quantities ω_{sr} (the sunrise hour angle without obstacles), t_{srw} and t_{ssw}. The reduced solar energy $H_{w,n}$ for the nth day of the year can be calculated by multiplying the solar radiation measured without obstacles H_n by the ratio ρ.

4. OPTIMUM SURFACE SLOPE

The solar energy incident on a sloped surface is higher than that on the horizontal plane, provided the slope and the orientation of the surface are properly adjusted. The optimum angle β_{opt} for the sloped surface, with zero azimuth angle, for a one month interval, is [8]:

$$\beta_{opt} = \arctan\frac{(1-L)a}{(1-L)b + (L - \rho_g)/2}, \quad (8)$$

where ρ_g is the ground reflection coefficient and the quantities a, b and L are given by the relations:

$$a = \frac{\tan\varphi\sin\omega_\beta - \frac{\pi\omega_\beta}{180}\tan\delta}{\sin\omega_{sr} + \frac{\pi\omega_{sr}}{180}\tan\varphi\tan\delta}$$

$$b = \frac{\sin\omega_\beta + \frac{\pi\omega_\beta}{180}\tan\varphi\tan\delta}{\sin\omega_{sr} + \frac{\pi\omega_{sr}}{180}\tan\varphi\tan\delta}$$

$$L = \begin{cases} 0.99 & \text{for } K_T \leqslant 0.17 \\ 1.188 - 2.272K_T + 9.473K_T^2 - 21.865K_T^3 + 14.648K_T^4 & \text{for } 0.17 < K_T \leqslant 0.75 \\ -0.54K_T + 0.632 & \text{for } 0.75 < K_T < 0.80 \\ 0.2 & \text{for } K_T \geqslant 0.80 \end{cases} \quad (9)$$

where K_T is the monthly average daily clearness index defined as $K_T = H/H_0$, H is the monthly average daily radiation [9], H_0 is the extraterrestrial monthly average daily radiation given in ref. [10] and ω_β is the sunrise solar hour angle on a surface with slope β. The relation that provides ω_β contains the unknown angle β_{opt}, thus an iterative numerical procedure is necessary for the computation of β_{opt}. However, with the following approximation the optimum angle is:

$$\omega_{sr} \approx \omega_\beta \Rightarrow b = 1 \Rightarrow \beta_{opt} = \arctan\frac{(1-L)a}{1-(L+\rho_g)/2}. \quad (10)$$

For better results, the slope of the surface must be adjusted sometimes during the course of a year. The calculation of the optimum slope over a period of i months is given by the following relation (applying the above approximation):

$$\beta_{opt} = \arctan\frac{\sum_i H(1-L)a}{\sum_i H[1-(L+\rho_g)/2]}. \quad (11)$$

The computation of the monthly average daily radiation on a tilted surface H_β when the slope of the surface is known is given in ref. [10]. Then, the clipped solar energy on a tilted surface $H_{\beta w}$ can be derived by multiplying the radiation without obstacles H_β by the ratio ρ calculated by eq. (7) for the mean day of the month.

5. DIMENSIONING

This method is proposed by Saha *et al.* [1] and is based on a very simple semi-empirical model for the computation of the PV array nominal power which must be installed for the accommodation of a specified load demand. This procedure does not take into account direct financial data. The rated power for the PV array [kW] is found to be:

$$P_{pv} = \frac{n_{pv}}{n_{ov}} \cdot \frac{12}{\sum_{m=1}^{12} H_{\beta w,m}} \cdot \left(E_L + \frac{E_L d_B}{d_c n_{BE}}\right). \quad (12)$$

In the above relation, n_{pv} is the PV array efficiency, n_{ov} is the overall efficiency of the system (PV array, DC/DC and DC/AC converters, transmission lines, etc.) $H_{\beta w,m}$ is the monthly average daily radiation on the array surface for the month $m = 1, 2, \ldots, 12$, E_L is the monthly average daily energy consumption of the load [kWh], d_B is the number of days for the autonomous operation of the load on the battery, d_c is the number of days required for the total battery charge to recover and n_{BE} is the energy efficiency of the battery. The maximum energy required to be stored in the battery is:

$$E_B = \frac{E_L d_B}{c_d}, \quad (13)$$

where c_d is the maximum permissible percentage depth of discharge of the battery.

In sites where the power grid is not available, a procedure is described in ref. [1] for an economic comparison between a PV system installation and a conventional generator. However, an accurate estimation of the cost is difficult because of the number of complex factors involved. For the economic analysis, the concept of the amortized annual operating expenses (AAOE) is used, i.e. the depreciation and interest on the initial capital investment combined with fuel cost, operator salary and maintenance. The AAOE for the PV system, e_{PVS}, is given by the relation:

$$e_{PVS} = \left(R + \frac{1}{D}\right)(C_F + C_P \cdot P + C_B + C_L) + S_F + S_P \cdot P, \quad (14)$$

where R is the rate of interest on the capital, D is the depreciation lifetime, C_F is the independent-of-power installation cost, C_P is the installation cost per kW, P is the installed rated power, C_B is the battery installation cost, C_L is the electronics (inverters, converters, regulators, etc.) installation cost, S_F is the fixed service cost (i.e. operator salary per year), and S_P is the operation and maintenance expenditure per year kW. The AAOE for a conventional power generator, e_G, is given by:

$$e_G = \left(R + \frac{1}{D}\right)(C_{FG} + C_{PG} \cdot P_G) + S_{FG} + S_{PG} \cdot P_G + F \cdot P_G. \quad (15)$$

The terms in relation (14), as in relation (15), are referred to the conventional generator. Here F is the annual fuel cost per kW of the generator. The rated power of the generator must be rationally reduced for continuous duty operation and the smallest commercially available sizes must be taken into account.

The comparison of the results of the two relations above can support the decision making between the installation of a PV system or a conventional generator, considering the economic view. Other factors such as the environmental impact, operating noise, fire danger, etc. could affect the final selection.

6. VERIFICATION

The verification procedure yields the possible number of days in a period of one year during which the load is not supplied by the PV system. The relation for the average daily efficiency of the PV array is [8]:

$$n_{pv} \cong n_{pvo}\left[1 - b_v(T_{avg} + 2 - T_r) - 1.25 a_t b_v \frac{H_n}{T}\right], \quad (16)$$

where a_t and b_v are temperature coefficients, T_{avg} is the daily average ambient temperature and T is the time interval between sunrise and sunset.

The available energy during one day, at the DC/DC converter output E_{dc} [kWh], is given by:

$$E_{dc} \approx 10 n_{pv} n_{dc} P_{pv} H_n. \quad (17)$$

Taking into account the maximum permissible depth of discharge of the battery c_d, the minimum permissible energy deposit of the battery is calculated by:

$$E_{Bmin} = (1 - c_d) E_B. \quad (18)$$

The usual type of load that has to be fed is lighting devices, which operate after sunset and before sunrise. Thus, it is assumed that during the day the PV array charges the battery, while during the night the load is supplied by the battery. The constraints for the charging and discharging of the battery must not be violated, $E_{Bmin} \leqslant E_b \leqslant E_B$, where E_b is the currently stored energy. The algorithm for the estimation of the possible days D without power at the load, on a yearly basis, is described below (starting with $D = 0$).

Step 1. Charging of the battery during the day, $E_{b,2} = E_{b,1} + E_{dc} n_B$. If $E_{b,2} > E_B$ then $E_{b,2} = E_B$. D is not changed.

Step 2. Discharging of the battery during the night, $E_{b,3} = E_{b,2} - E_{dc}/n_B$. If $E_{b,3} < E_{Bmin}$ then $E_{b,3} = E_{Bmin}$ and $D = D+1$. Return to Step 1 for the second day's test.

The above procedure is performed for all 365 days of the year.

7. APPLICATION EXAMPLE

The above algorithm was employed in the case of the Samaria gorge, on the island of Crete. A PV system for the electricity demand of a small observatory in the middle of the gorge was studied. The utility grid is unavailable inside the gorge and the walls considerably obstruct the solar radiation reaching the bottom.

The load consists of two lamps of 9 W each, a radio communications device of 5 W, a charger of 2 W and an alarm device of 10 W. The average daily load energy is then calculated to be 0.351 kWh. The observed sunrise and sunset times were 9:00 and 16:00 in May and 07:55 and 12:55 in October, respectively.

The rate of interest is set to 25%, the depreciation lifetime is considered to be 10 years, the PV array cost per kW is \$8500, the fixed cost is \$200, and the battery cost per kWh is \$105. The rates for a 2 kW conventional gasoline generator are: cost per kW = \$200, fixed cost = \$200, fixed service cost = \$200, fuel consumption per hour = 0.1 l, with 10 hours of operation per day.

The sunrise and sunset times (LAT) with and without obstacles at the bottom of the gorge are shown in Table 1. The average solar radiation for each month on the horizontal H, at the bottom of the gorge H_w and on the sloped surface of the PV array $H_{\beta w}$ are shown in Table 2.

The results indicate that a change of tilt twice a year gives almost the same size of the PV system as if one constant tilt (the higher of the two optimal slopes) is used. This is because the load has to be mainly supplied during the night-time period and is almost constant during the year, thus the lowest solar radiation month (December) affects the computations. The results of the computer program are as follows.

Optimal slopes: 6.94° and 48.63°
PV array rated power: 0.3 kW
Battery capacity: 2 kWh
Number of days with no power: 22
Annual amortized operating expenses for PV: \$1093
Annual amortized operating expenses for generator: \$1422

8. CONCLUSIONS

The annual amortized cost of the PV array is lower than that of the conventional generator, for the application studied above, indicating that the PV array is a cost-effective solution.

Obstacles regarding the type of gorge walls can reduce the solar energy considerably, thus the calculation of their impact for optimal dimensioning is necessary. The proper slope of the PV array improves the efficiency of the system, especially during the winter months when the use of the system is more vital.

The verification routine is reliable if tested over a period of at least 10 years. For the case studied here, only 1 year of solar radiation data were available, which justifies the above results. It is proposed to use the same slope for the whole year, the higher one from the two optimal slopes calculated by the computer program.

Table 1. Sunrise and sunset hours with and without obstacles

	Sunrise		Sunset			Sunrise		Sunset	
Month	w/o obstacles	with obstacles	w/o obstacles	with obstacles	Month	w/o obstacles	with obstacles	w/o obstacles	with obstacles
1	07:02:26	07:44:05	16:57:33	11:56:22	7	04:46:39	07:17:48	19:03:20	14:50:39
2	06:37:17	07:44:58	17:22:42	12:31:36	8	05:21:12	07:47:45	18:38:47	14:16:13
3	06:06:48	07:46:05	17:53:11	13:13:32	9	05:53:44	07:46:34	18:06:15	13:31:26
4	05:33:10	07:47:19	18:26:49	13:59:40	10	06:27:21	07:45:20	17:32:38	12:45:20
5	05:04:33	07:39:29	18:55:26	14:39:29	11	06:55:49	07:44:19	17:04:10	12:05:43
6	04:50:08	06:58:47	19:09:51	14:59:56	12	07:09:43	07:43:49	16:50:16	11:45:59

Table 2. Radiation in kWh/m^2

Month	H	H_w	$H_{\beta w}$	Month	H	H_w	$H_{\beta w}$
1	2.390	1.140	1.511	7	7.690	5.891	4.566
2	2.400	1.335	1.385	8	6.750	4.779	4.228
3	4.060	2.559	2.676	9	5.340	3.503	3.684
4	5.780	3.992	3.726	10	3.680	2.145	2.562
5	6.610	4.877	3.998	11	3.110	1.554	2.353
6	7.660	6.035	4.551	12	1.910	0.860	1.055

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