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SIZE OPTIMIZATION OF A PV SYSTEM INSTALLED CLOSE TO SUN OBSTACLES

K. KALAITZAKIS and G. S. STAVRAKAKIS
Technical University of Crete, Electronics and Computer Engineering Department, GR 731 00 Chania, Greece

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Abstract—The assessment of the optimal size of a PV-array/battery-storage system, when the incident solar radiation is considerably obstructed, is presented in this article. An optimal dimensioning method is proposed based on a new procedure for the calculation of the actual solar radiation on the array surface, for installations located near obstacles. The optimum tilt of the PV array is also computed and the resulting optimal PV-array/battery-storage system is evaluated by an appropriate routine. The results of the application of the complete algorithm for a real case study inside a gorge are illustrated. Copyright © 1997 Elsevier Science Ltd.

1. INTRODUCTION

The relatively high cost of both the photovoltaic (PV) array and the battery storage necessitates the development of a reliable method for the estimation of their size, thus minimizing the investment cost of the installation, for isolated electric consumers powered by photovoltaic devices when the utility grid is unavailable. Many approaches to this dimensioning problem for clear flat terrains, based mainly on how to minimize the fraction of a given load not covered at the lowest cost, have been proposed (Barra *et al.*, 1984; Saha *et al.*, 1988; Chapman, 1989; Elalaoui Faris, 1990; Abouzahr and Ramakumar, 1991; Gavanidou and Bakirtzis, 1992). Some investigators have proposed methods for the load matching and management, whenever possible, to reduce the PV size (Groumbos and Papageorgiou, 1991; Khouzam *et al.*, 1991).

The optimal sizing is more necessary for installations where the solar radiation incident on the PV surface is considerably limited by obstacles that cannot be avoided. One such application is the installation of a PV array on the bottom of a gorge or a valley or on the surface of buildings shaded by neighboring buildings. An analytical method is proposed for the computation of the optimal dimensions of the system. The solar radiation and the PV system data are required as input. The output is the optimal PV-array rated power and the battery-storage capacity.

A method for the computation of the real available solar radiation on the PV panel is proposed here. Limited measurements of the incident solar energy radiation on the location of the installation are used and the necessary concepts for the solar calculations are mentioned. Another factor playing an important role on the size of the system is the tilt of the PV-array surface to the horizontal plane. A consistent method for the computation of one or two optimal slopes of the PV surface is proposed here. The battery storage increases generally the reliability of the system, whereas in some cases it is unavoidable.

A procedure for the evaluation of the sufficiency of the above computed PV system improves the reliability of the method, taking into account the load demand and the solar radiation data measured at the location of the system. All the above procedures build a complete algorithm performing the dimensioning with the least possible effort from the user.

An application example is finally illustrated for a PV system located at the bottom of the Samaria gorge in the island of Crete, Greece, supplying an observatory.

2. THE OPTIMAL DIMENSIONING METHOD

In order to select the optimal size of the PV array/battery storage a method first proposed in Barra *et al.* (1984) is presented here in brief in a revised and improved form.

The following assumptions are considered.

(1) The daily PV array production is proportional to the global solar irradiation

received on the array surface, thus the PV array is characterized completely by its active surface A (m^2) and its efficiency n_{pv}.

(2) The efficiency of the battery storage is the same during battery charging and discharging, thus the battery storage is completely characterized by its capacity E_B (kWh) and its efficiency mean value n_B, considered as constant during the system steady state operation (normally it depends on the charging and discharging current, so it fluctuates during the system operation).

(3) The load mean value is considered constant during the examined period (it can change during the day, but its daily mean value remains constant during the examined period).

(4) The other electronic devices (DC/DC, DC/AC converters) have a constant efficiency n_{dc} (or n_{ac}) during the system operation.

(5) The overall amount of energy produced by the PV array is stored in the battery from which the load is served.

Let one define the quantity Z representing the average fraction of the load not covered by the PV system during the month with the worst solar radiation, thus Z represents an indication of the loss of load probability (LOLP) for the examined period (i.e. the month with the worst solar radiation).

Let one define as X the maximum energy amount arriving at the load for the examined period, normalized on the load. Thus one can write: $X = AH_{\beta w}n_{pv}n_{dc}n_B n_{ac}/E_L$, where $H_{\beta w}$ is the clipped solar energy on a tilted surface with tilt angle β (kWh/m^2) and E_L is the monthly average daily energy consumption of the load (kWh).

The battery storage capacity normalized to the load is given by the relation: $W = E_B K_T n_B d/E_L$, where K_T is the monthly average daily clearness index and $d = T_j/24$, T_j being the mean time from the sunrise to the sunset (h).

Therefore, the average fraction of the load covered by the PV system during the examined period is represented by $Y = 1 - Z$, thus $0 \leq Y \leq 1$. Obviously, the following conditions must be valid for Y.

(1) When the PV is under-dimensioned w.r.t. the load: $X \to 0 \Rightarrow Y \to 0$ thus $Z \to 1$.

(2) When the PV is over-dimensioned w.r.t. the load: $X \to 1 \Rightarrow Y \to 1$ thus $Z \to 0$.

(3) When $W \geq 1 \Rightarrow Y = 1$.

The explicit analytical form for Y derived from the above analysis and the considerations of Barra *et al.* (1984) is the following hyperbolic equation: $(X - Y)(1 - Y) = aW^{-b}$, where a and b are appropriate constants which are derived by a least squares fitting method using laboratory measurements on concrete PV arrays with known A, E_B and E_L sizes (Barra *et al.*, 1984; Elalaoui Faris, 1990), in such a way that the above hyperbolic equation fits with a minimal error with these measurements.

In Barra *et al.* (1984) and Elalaoui Faris (1990) using a big amount of simulations and comparisons with real measurements from various PV arrays–battery storage operation it is proposed to take (a) $a = 6.6 \times 10^{-3}$ and $b = 3.4$ for consumptions uniformly distributed between day and night and (b) $a = 5.26 \times 10^{-2}$ and $b = 5$ for consumption increased during the night (i.e. increased necessity for battery storage capacity). Let us introduce the following normalized quantities:

$$x = \frac{X}{A} = \frac{H_{\beta w}n_{pv}n_{dc}n_B n_{ac}}{E_L} \quad \text{and} \quad w = \frac{W}{E_B} = \frac{K_T n_B d}{E_L}$$

thus the above hyperbolic equation of Barra *et al.* (1984) becomes

$$(Ax - Y)(1 - Y) = aw^{-b}E_B^{-b}$$

and by solving it with respect to A, one obtains

$$A = \frac{Y}{x} + \frac{aw^{-b}E_B^{-b}}{(1 - Y)x}$$

The optimum design must minimize the global cost of the installation. If C_{pv} is the cost of the PV array in (\$/m^2) and C_B is the battery storage cost in (\$/kWh), thus the total cost of the installation PV array–battery storage is expressed as a function only of E_B by the equation

$$C_t = C_{pv}A + C_B E_B = C_{pv}\left[\frac{Y}{x} + \frac{aw^{-b}E_B^{-b}}{(1 - Y)x}\right] + C_B E_B \tag{1}$$

If one finds the minimum of eqn (1) w.r.t. E_B, i.e. $dC_t/dE_B = 0$, one obtains

$$E_{B,opt} = \left[\frac{C_{pv}abw^{-b}}{C_B(1 - Y)x}\right]^{\frac{1}{1+b}} \tag{2}$$

It is noted that the quantities C_{pv}, C_B, Y, x, w, a, b are by definition independent from the quantity E_B. In practice it is usually known the ratio of the PV array cost w.r.t. the battery storage cost, i.e. $c = C_{pv}/C_B$, thus eqn (2) can be written as

$$E_{B,opt} = \left[\frac{abcw^{-b}}{xZ}\right]^{\frac{1}{1+b}} \quad (3)$$

By substituting eqn (3) into the equation for A derived previously, one gets the optimal PV array size as

$$A_{opt} = \frac{Y}{x} + \left[\frac{a(bcw)^{-b}}{xZ}\right]^{\frac{1}{1+b}} \quad (4)$$

An equivalent graphical solution of the optimal dimensioning problem, very useful in the engineering practice, can also be taken if the analytic expression of the installation characteristics A/E_B is derived and plotted.

By taking the derivative of A w.r.t. E_B, one can obtain the expression (Elalaoui Faris, 1990)

$$D \equiv \frac{dA}{dE_B} = -b\frac{aw^{-b}}{xZ}E_B^{-(1+b)}$$

$$\Rightarrow \frac{aw^{-b}}{xZ}E_B^{-b} = -\frac{DE_B}{b} \quad (5)$$

thus it can be derived that

$$\frac{A}{E_B} = \frac{Y}{xE_B} - \frac{D}{b}$$

By solving eqn (5) w.r.t. E_B and by substituting the resulting expression for E_B in the above equation, the following equation is obtained:

$$\frac{A}{E_B} = -\frac{D}{b} + \frac{Y}{x}\left[-\frac{D}{b\frac{aw^{-b}}{xZ}}\right]^{\frac{1}{1+b}} \quad (6)$$

thus using also the expressions for x and w, the following equation is finally obtained:

$$\frac{A}{E_B} = -\frac{D}{b} + \left(-\frac{D}{ab}\right)^{\frac{1}{1+b}} Y(1-Y)^{\frac{1}{1+b}} \left(\frac{H_{\beta w}n_{pv}n_{dc}n_{ac}}{K_T d}\right)^{-\frac{b}{1+b}} \quad (7)$$

The $A \rightarrow E_B$ characteristics (nomograms) described by eqn (7) are shown in Fig. 1 for different values of Y or the LOLP equivalent Z, for the given solar radiation, climate conditions and the consumption requirements of the specific case.

The PV array–battery storage cost ratio of the installation is given by $c_o = C_{pv}A/C_BE_B = c(A/E_B)$. Given that c is usually a constant easy to calculate from the mean values of C_{pv} and C_B taken from the market, the technical–economic optimal dimensioning of the installation is given by the points where the iso-cost lines are tangent to the (A/E_B) characteristics, i.e. the bend region of Fig. 1. PV-array manufacturers could elaborate and provide the curves of Fig. 1, for different sites or for a variety of solar radiation conditions, in order to facilitate the PV array–battery storage optimum selection by a simple visual inspection of these nomograms.

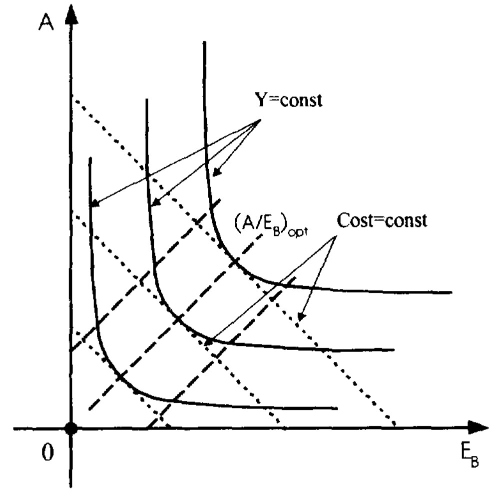

Fig. 1. PV area A vs. battery storage E_B, for some values of Z.

3. THE ACTUAL SOLAR RADIATION ON THE PV ARRAY

There are many PV installations where unavoidable obstacles limit the solar radiation on the PV surface (i.e. a PV array at the bottom of a gorge or a narrow valley or on the surface of buildings shaded by neighboring buildings), considerably reducing the total energy incident to the PV array during a day period. A new method for the calculation of the impact of obstacles on the solar energy actually captured by the PV array is proposed here.

Let d be the distance of the PV array (located at A) from the obstacle, h the height of the obstacle, and γ_f the angle between its direction and the north–south axis, as shown in Fig. 2. The ratio $r = d/h$ is constant for the point A and with simple trigonometric calculations the following relation is derived:

$$r \equiv \frac{d}{h} = \sin(\gamma_f - \gamma_s)\tan\theta_z \quad (8)$$

where θ_z is the zenith angle and γ_s is the solar

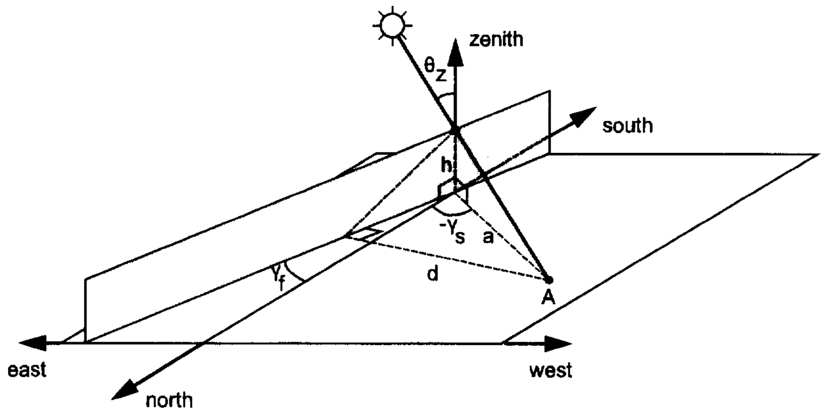

Fig. 2. Sunrise hour angle change by an obstacle.

azimuth angle (Duffie and Beckman, 1994), when the first solar beam reaches the observer at point A, given by the relations

$$\sin\gamma_s = \frac{\cos\delta \sin\omega}{\sin\theta_z} \tag{9a}$$

$$\omega = 15°(t-12) \tag{9b}$$

$$\cos\theta_z = \sin\phi \sin\delta + \cos\phi \cos\delta \cos\omega \tag{9c}$$

$$\delta = 23.45° \sin\left[360\frac{284+n}{365}\right] \tag{9d}$$

$$t_{\mathrm{LAT}} = t_{\mathrm{LST}} + T_e - 2 + \frac{\lambda}{15°}[-1] \tag{9e}$$

$$T_e = 0.0072\cos\left(\frac{2\pi n}{365}\right) - 0.0528\cos\left(\frac{4\pi n}{365}\right) - 0.0012\left(\frac{6\pi n}{365}\right) - 0.1229\sin\left(\frac{2\pi n}{365}\right) - 0.1565\sin\left(\frac{4\pi n}{365}\right) - 0.0041\sin\left(\frac{6\pi n}{365}\right) \tag{9f}$$

where ω is the solar hour angle, ϕ is the latitude, t is the time of the day, expressed as local apparent time (LAT) and δ is the solar declination for the nth day of the year (Duffie and Beckman, 1994). Relation eqn (9e) gives the local apparent time t_{LAT} as a function of the local standard time t_{LST} and the day of the year n. The [−1] is activated only during the summer period (March–September) because, in that interval, the local standard time is increased by 1 h, in most countries. The delayed sunrise time t_{srw} for any day of the year can be computed following the next steps.

(1) Measure the quantities d, h and γ_f. Calculate the ratio $r = d/h$ and the declination δ for the specified day. The latitude ϕ is considered known for the site.
(2) The numerical solution of the simultaneous eqns (8), (9a) and (9c) yields the quantities γ_s, θ_z and ω_{srw}.
(3) From eqn (9b) calculate t_{srw}.

Although in many applications the characteristics of the obstacle (r and γ_f) are not known or are difficult to measure. If this is the case, two observations of the sunrise time (at the site A) for any two different days of the year are required. If one observes the sunrise time for only two different days of the year as t_{sr1} and t_{sr2}, the two pairs (θ_{z1}, γ_{s1}) and (θ_{z2},γ_{s2}) can be computed by eqns (9a), (9b), (9c), (9d), (9e) and (9f). Then relation eqn (8) results in the following equation:

$$\sin(\gamma_f - \gamma_{s1})\tan\theta_{z1} = \sin(\gamma_f - \gamma_{s2})\tan\theta_{z2} \tag{10}$$

Equation (10) can be solved for γ_f by any appropriate numerical method (i.e. secant root finding method). Using this value of γ_f the value of r can be calculated by eqn (8). The same procedure can be performed if an obstacle affects the sunset time. If this is the case, the term ($\gamma_f - \gamma_s$) must be replaced by ($\gamma_s - \gamma_f$) in all the above relations.

The total solar irradiation during any day $G(t)$ (kW/m²), from sunrise t_{sr} to sunset t_{ss} is described by the curve of Fig. 3, where the corresponding sunrise and sunset times t_{srw} and t_{ssw} for that day due to the presence of obstacles are calculated as described above.

The relation between the radiation and the

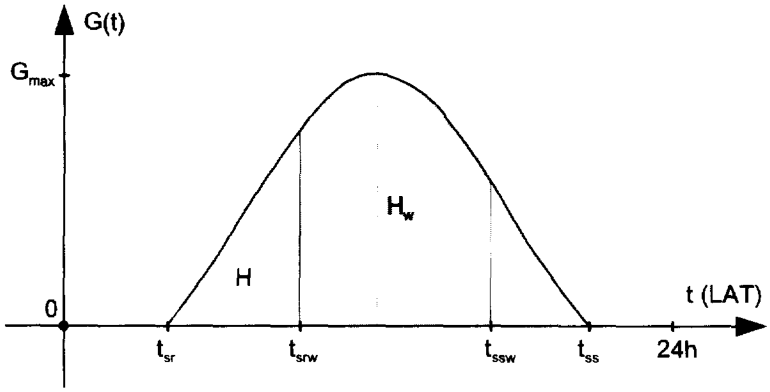

Fig. 3. Daily solar radiation between obstacles.

solar hour angle is

$$G(t)=M\left[b_M\cos^2\left(\frac{\pi t}{12}\right)-\left(a_M-b_M\cos\omega_{sr}\right)\times\cos\left(\frac{\pi t}{12}\right)-a_M\cos\omega_{sr}\right] \tag{11}$$

with

$$M=\frac{\pi H_n}{24\left(\sin\omega_{sr}-\dfrac{\pi\omega_{sr}}{180}\cos\omega_{sr}\right)} \tag{12a}$$

$$\omega_{sr}=\arccos(-\tan\phi\tan\delta) \tag{12b}$$

$$a_M=0.409+0.5016\sin(\omega_{sr}-60°) \tag{12c}$$

$$b_M=0.6609-0.4767\sin(\omega_{sr}-60°) \tag{12d}$$

where H_n (kWh/m^2) is the measured total solar radiation during the nth day of the year. The result of integration of eqn (11) is the solar energy in the integration interval. The shaded portion of Fig. 3 represents the available daily solar energy $H_{w,n}$ if some obstacles shift the sunrise and sunset times to t_{srw} and t_{ssw}, respectively.

After some calculations, the ratio of the daily solar radiation with and without obstacles is found to be, for the nth day of the year,

$$\rho_n=\frac{H_{w,n}}{H_n}=\frac{\pi}{24\left(\sin\omega_{sr}-\dfrac{\pi\omega_{sr}}{180}\cos\omega_{sr}\right)}\times\left[\left(\frac{b_M}{2}-a_M\cos\omega_{sr}\right)(t_{ssw}-t_{srw})-\frac{12(a_M-b_M\cos\omega_{sr})}{\pi}\times\left(\sin\left(\frac{\pi t_{ssw}}{12}\right)-\sin\left(\frac{\pi t_{srw}}{12}\right)\right)+\frac{3b_M}{\pi}\left(\sin\left(\frac{\pi t_{ssw}}{6}\right)-\sin\left(\frac{\pi t_{srw}}{6}\right)\right)\right] \tag{13}$$

It becomes obvious that the ratio ρ is a function of the known quantities ω_{sr} (the sunrise hour angle without obstacles), t_{srw} and t_{ssw}. The reduced solar energy $H_{w,n}$ for the nth day of the year can be calculated multiplying the solar radiation measured without obstacles H_n by the ratio ρ_n.

The solar energy incident on a sloped surface is higher than that on the horizontal plane, provided the slope and the orientation of the surface is properly adjusted. For better results, the slope of the surface must be adjusted sometimes in a year period. Thus, the calculation of the optimum slope over a period of a specific month of the year must be determined. The monthly average daily radiation incident on a surface tilted by an angle β, H_β, is given by (Klein, 1977; Duffie and Beckman, 1994) the equation

$$H_\beta=R\cdot H \tag{14}$$

where H is the monthly average daily radiation measured at the specified site for a period of at least 1 year and R is the ratio of the total radiation on the tilted surface to the total radiation on the horizontal and is given by the equation

$$R=\left(1-\frac{H_d}{H}\right)\cdot R_b+\frac{H_d}{H}\cdot\frac{(1+\cos\beta)}{2}+\rho_g\cdot\frac{(1-\cos\beta)}{2} \tag{15}$$

where H_d is the diffused monthly average daily radiation, ρ_g is the ground reflection coefficient (between 0.2 and 0.7) and R_b is calculated by the equation

$$R_b = \frac{\cos(\phi-\beta)\cos\delta\sin\omega_\beta + \frac{\pi\omega_\beta}{180}\sin(\phi-\beta)\sin\delta}{\cos\phi\cos\delta\sin\omega_{sr} + \frac{\pi\omega_{sr}}{180}\sin\phi\sin\delta} \quad (16)$$

where ω_β is the sunrise solar hour angle on the tilted surface, given by the equation

$$\omega_\beta = \min[\omega_{sr}, \arccos(-\tan(\phi-\beta)\tan\delta)] \quad (17)$$

Setting $\partial H_\beta/\partial\beta = 0$ and using eqns (14)–(17), the following relation for the optimal tilt β_{opt}, is obtained:

$$\beta_{opt} = \arctan\frac{(1-L)a_L}{(1-L)b_L + (L-\rho_g)/2} \quad (18)$$

where the quantities a_L, b_L and $L = H_d/H$ are given by the relations

$$a_L = \frac{\tan\phi\sin\omega_\beta - \frac{\pi\omega_\beta}{180}\tan\delta}{\sin\omega_{sr} + \frac{\pi\omega_{sr}}{180}\tan\phi\tan\delta} \quad (19a)$$

$$b_L = \frac{\sin\omega_\beta + \frac{\pi\omega_\beta}{180}\tan\phi\tan\delta}{\sin\omega_{sr} + \frac{\pi\omega_{sr}}{180}\tan\phi\tan\delta} \quad (19b)$$

$$L = \begin{cases} 0.99 & \text{for } K_T \leq 0.17 \\ 1.188 - 2.272K_T \\ +9.473K_T^2 - 21.865K_T^3 \\ +14.648K_T^4 & \text{for } 0.17 < K_T \leq 0.75 \\ -0.54K_T + 0.632 & \text{for } 0.75 < K_T < 0.80 \\ 0.2 & \text{for } K_T \geq 0.80 \end{cases} \quad (19c)$$

where K_T is the monthly average daily clearness index defined as $K_T = H/H_o$, H_o is the extra-terrestrial monthly average daily radiation for any site, given by (Duffie and Beckman, 1994) the equation

$$H_o = 37{,}210{,}171 \cdot \left[1 + 0.033\cos\left(\frac{2\pi n_m}{365}\right)\right] \cdot \left[\cos\phi\cos\delta\sin\omega_{sr} + \frac{\pi\omega_{sr}}{180}\sin\phi\sin\delta\right] \quad (20)$$

where n_m is the mean day of the month, given by Duffie and Beckman (1994). The relation eqn (17) that provides ω_β contains the unknown angle β_{opt}, thus an iterative numerical procedure is necessary for the computation of β_{opt}. However, with the following approximation the optimum angle is

$$\omega_{sr} \approx \omega_\beta \Rightarrow b_L = 1 \Rightarrow \beta_{opt} = \arctan\frac{(1-L)a_L}{1-(L+\rho_g)/2} \quad (21)$$

The error of the β_{opt} caused by the approximation is found to be $\pm 1.5°$ (Balouktsis *et al.*, 1987) thus the error on the incident radiation is negligible. The optimal slope for a period of i months is derived with the same procedure and is found to be

$$\beta_{opt} = \arctan\frac{\sum_i (1-L)a_L}{\sum_i [(1-L)b_L + (L-\rho_g)/2]} \quad (22)$$

4. EVALUATION OF THE DIMENSIONING RESULTS

The results of the above method for the PV array and the battery storage size must be verified using available solar radiation data. The proposed method estimates the possible number of days, in any 1-year period, during which the load will not be supplied by the PV system.

The available energy during 1 day, at the DC/DC converter output E_{dc} (kWh), is given by

$$E_{dc} = n_{pv} n_{dc} A H_n \quad (23)$$

where n_{pv} is the average daily efficiency of the PV array and is given by the following well-known relation (Balouktsis *et al.*, 1987):

$$n_{pv} \cong n_{pvo}\left[1 - b_v(T_{avg} + 2 - T_r) - 1.25a_t b_v \frac{H_n}{T}\right] \quad (24)$$

where a_t and b_v are temperature coefficients,

T_{avg} is the daily average ambient temperature and T is the time interval between sunrise and sunset.

Taking into account the maximum permissible depth of discharge of the battery c_d, the minimum permissible energy deposit of the battery is calculated by

$$E_{Bmin} = (1 - c_d)E_B \quad (25)$$

The more usual type of load that has to be fed is lighting devices, which operate after the sunset and before the sunrise. Thus, it is assumed that during the day the PV array charges the battery, while during the night the load is supplied by the battery. The constraints for the charging and discharging of the battery must not be violated, $E_{Bmin} \leq E_b \leq E_B$, where E_b is the currently stored energy. The algorithm for the estimation of the possible days D_L without power at load, in any 1-year basis, is described below (starting with $D_L = 0$).

(1) Step 1. Charging of the battery during the day, $E_{b,2} = E_{b,1} + E_{dc} n_B$. If $E_{b,2} > E_B$ then $E_{b,2} = E_B$. D_L is not changed.
(2) Step 2. Discharging of the battery during the night, $E_{b,3} = E_{b,2} - E_{dc}/n_B$. If $E_{b,3} < E_{Bmin}$ then $E_{b,3} = E_{Bmin}$ and $D_L = D_L + 1$. Return to Step 1 for the second day test.

The above procedure is performed for all 365 days of the year.

5. DESCRIPTION OF THE COMPLETE ALGORITHM

The methodology described above has been integrated into a comprehensive algorithm, on which a computer program is based. The successive steps of the procedure are described below.

(1) The measured monthly average daily radiation H for a year period (12 values), longitude λ and latitude ω for the specified site, along with the table for the mean day of each month n_m (Duffie and Beckman, 1994), are used as input for the calculation of the monthly average values of declination δ (eqn (9d)), extraterrestiral radiation H_o (eqn (20)), clearness index K_T and sunrise t_{sr} and sunset t_{ss} times (eqn (9b) and eqn (12b)) for each month, on the horizontal plane. The times involved in all equations are in local apparent time form. The conversion formulas from the standard time to LAT are given in eqn (9e) and eqn (9f).
(2) The observed sunrise time t_{srw} at the specified site, for two different months of the year is required for the calculation of θ_z and γ_s (eqn (9a) and eqn (9c)) for each month. Then, solving eqn (10) for γ_f, the angle of the obstacle to the north–south axis is derived. The observations are preferred to be away from each other (no successive months) for better accuracy. The standard time indications must be converted to LAT ones, as described above. If there are obstacles that affect the sunset time t_{ssw}, the above procedure is repeated for the other obstacle (i.e. wall of the gorge). Then, the ratio r for each wall is computed by means of eqn (8). Equations (8), (9a) and (9c) are solved simultaneously, providing the sunrise (sunset) solar hour angle ω_{srw} or the sunrise time t_{srw} (eqn (9b)) between the walls, for each month.
(3) The above results are used in eqn (13) in combination with eqns (12a), (12b), (12c) and (12d) and the ratio ρ for each month is derived. Then, the monthly average daily radiation H_w, the extraterrestrial radiation H_{ow} and the clearness index K_{Tw} between the walls are the products of the original values and ρ.
(4) Using the previous computed values for H_{ow}, K_{Tw} and the ground reflection coefficient ρ_g, together with eqns (17), (19a), (19b) and (19c) and (22), over a period of 12 months, the optimal slope β_{opt} for the PV array between the walls can be elaborated. If two slopes are required for better performance, eqn (22) must be applied for each of the April–September and October–March periods.
(5) The optimal slopes are used for the computation of the monthly average daily radiation on the tilted surface between the walls $H_{\beta w}$. Equations (14)–(17) and (19c) are used for this purpose.
(6) The sizing method uses as input the efficiencies of the PV array n_{pv}, the DC/DC converter n_{dc}, the batteries n_B, together with the average daily load demand E_L, the loss of load probability Z, the cost ratio c, and the coefficients a and b. The result is the active area A of the PV array and the capacity E_B of the batteries, given by eqns (2) and (3).
(7) Finally, the evaluation procedure is used for the estimation of the reliability of the system. The average value of PV array

efficiency n_{pv}, or an improved value by means of eqn (24) can be used for the calculation of the available energy E_{dc} from eqn (23). The discharge depth of the battery is used and the minimum permissible energy deposit E_{Bmin} is derived (eqn (25)). The two steps of the procedure are executed next, and the number of days without power supply D_L is calculated.

6. APPLICATION RESULTS

The case of a PV system for the power supply of a small observatory in the middle of the 17-km long Samaria gorge on the south-west part of Crete, is studied here. The utility grid is unavailable in the gorge and the two sides of the gorge considerably prevent the solar radiation reaching the bottom. The necessary computations of radiation measurements for the site are found in PPC (1982).

The observed sunrise and sunset local standard times were 09:00 (07:39 when converted to LAT) and 16:00 (14:39 converted to LAT) in May and 09:45 (09:35 converted to LAT) and 14:15 (14:05 converted to LAT) in October, respectively. The load consists of 11 lamps of 9 W each, a radio communications device of 45 W, a charger of 2 W, an alarm device of 10 W, a refrigerator of 150 l with daily consumption of 0.5 kWh and a TV set with daily consumption of 0.1 kWh. The average daily load energy is then calculated to be 1.775 kWh.

The acceptable LOLP in the present case is set equal to 4%, i.e. $Z=0.004$, in order to assure 2 days of minimum autonomy of the installation in 98% of the studied cases. The ratio of the cost of the PV array per area (\$/m^2) to the cost of the battery per kWh (\$/kWh) is calculated to approximately 9, taking into account the present market prices. The a and b coefficients are estimated as described in Section 2.

The sunrise and sunset times (LAT) without and with obstacles at the bottom of the gorge, calculated by the proposed procedure, are shown in Table 1. The average solar radiation for each month on the horizontal H, at the bottom of the gorge H_w and on the sloped surface of the PV array $H_{\beta w}$ are shown in Table 2.

The algorithm presented in Section 5 for the computation of the optimal PV-array slope, the sizing of the PV array and the days with loss of load gives the following results. Optimal slopes, 6.94 and 48.63°; PV-array surface area, 17.6 m^2; battery capacity, 91.6 kWh; number of days with no power supply, 6.

Table 1. Sunrise and sunset hours (LAT) without and with obstacles

Month	Sunrise w/o obstacles	Sunrise with obstacles	Sunset w/o obstacles	Sunset with obstacles
1	07:02:26	10:28:14	16:57:33	13:50:44
2	06:37:17	09:49:58	17:22:42	14:01:01
3	06:06:48	09:04:44	17:53:11	14:13:41
4	05:33:10	08:15:00	18:26:49	14:27:42
5	05:04:33	07:39:29	18:55:26	14:39:29
6	04:50:08	07:34:49	19:09:51	14:45:18
7	04:56:39	07:36:55	19:03:20	14:42:41
8	05:21:12	07:57:06	18:38:47	14:32:39
9	05:53:44	08:45:27	18:06:15	14:19:08
10	06:27:21	09:35:08	17:32:38	14:05:08
11	06:55:49	10:18:03	17:04:10	13:53:25
12	07:09:43	10:39:35	16:50:16	13:47:48

Table 2. Radiation in kWh/m^2 (PPC, 1982)

Month	H	H_w	$H_{\beta w}$
1	2.390	1.271	1.685
2	2.400	1.442	1.495
3	4.060	2.690	2.812
4	5.780	4.117	4.200
5	6.610	4.877	4.872
6	7.660	5.649	5.580
7	7.690	5.672	5.631
8	6.750	4.905	4.972
9	5.340	3.651	3.806
10	3.680	2.293	2.739
11	3.110	1.717	2.600
12	1.910	0.970	1.190

Appropriate computer executions of the proposed procedure indicate that the change of the tilt twice a year gives almost the same size of the PV system if one constant tilt (the higher of the two optimal slopes) is used. This is because the load has to be mainly supplied during the night period and is almost constant during the year, thus the lowest solar radiation month (December) affects the computations.

The same case was studied using the sizing method described in Saha *et al.* (1988). The results follow: optimal slopes, 6.94 and 48.63°; PV-array surface area, 13.8 m^2; battery capacity, 10.2 kWh; number of days with no power supply, 20.

If the effect of the obstacles is not taken into account, the reliability of the system is substantially degraded. For the next case, the same site characteristics are used, but in the sizing procedure of Section 2, instead of the clipped radiation $H_{\beta w}$, the unprevented radiation on a sloped surface H_β is used. Optimal slopes, 6.94 and

48.63°; PV-array surface area, 8.6 m^2; battery capacity, 32 kWh; number of days with no power supply, 65.

The resulting system is quite smaller than the previous one, but the days with no power supply make it unacceptable.

7. CONCLUSIONS

The dimensioning method gives reasonable results and compared with other dimensioning methods (Saha *et al.*, 1988; Chapman, 1989; Gavanidou and Bakirtzis, 1992) is rather conservative, suggesting a higher battery-storage size and a slightly higher PV-array area. However, the number of days with no supply is substantially lower than that of other methods.

It must be noted here that the verification routine is reliable if applied over a period of at least 10 years. For the case studied here only 1 year solar radiation data were available. This justifies the above results. The slope is proposed to be unique for the whole year, the higher one from the two optimal slopes calculated by the computer program.

The strong difficulty and its additional cost to bring the fuel quantities needed in the case that a generator would be adopted instead of a PV system must also be taken seriously into account for installations like those of the present case study.

In the case of multiple obstacles, the solution of the problem is not generic but strongly application dependent, i.e. according to the relative position of the obstacles, their surfaces, etc. A good idea could be to consider the most significant obstacle and to perform the computations with respect to it.

The protection of the system from both faults and environmental conditions (lightning, fire, strong winds, high humidity, high temperature, etc.) must also be considered in applications like the isolated site of the Samaria gorge (Nailen, 1991).

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