Topology optimization of polymorphic structures and mechanisms using global and multicriteria optimization

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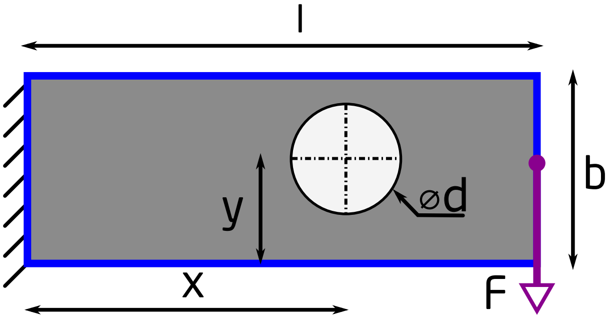
October 28, 2015

Overview

Design Optimization



- hole placement: x,y = ?
- hole diameter: d = ?
- one or more holes?
- circular hole or any other shape?
- material used?

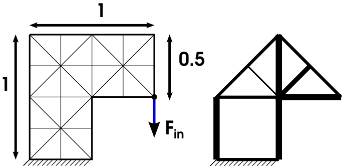
Structural Optimization: by Galileo Galilei - Discorsi (1638)

(a) Galileo's original cantilever beam's shape

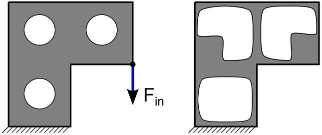
(b) Galileo's parabolic cantilever beam shape

Figure – Galileo's Shape Optimization

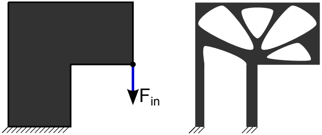
Categories of structural optimization



(a) Size optimization



(b) Shape optimization



(c) Topology optimization

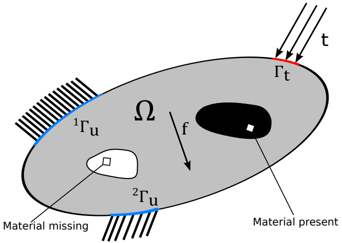
Figure – Different types of Structural Optimization

Topology optimization

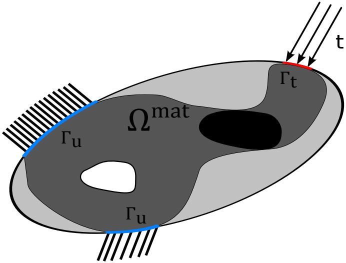
Definition

It is a mathematical method that optimizes material distribution inside a predefined design domain. The resulted structure must form a continuous body and sustain the applied loads and satisfies any boundary conditions.

Topology optimization: problem definition



(a) Design domain definition



(b) Resulted material distribution

Topology optimization: Problem definition

Objective function: Compliance

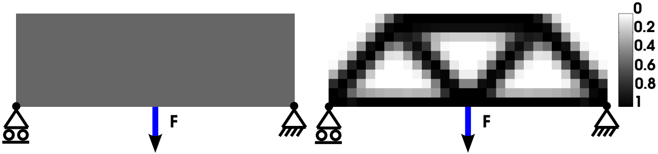
$$\min c = \mathbf{f}^T \mathbf{u} \tag{1}$$

State function: Equilibrium

$$\mathbf{f} = \mathbf{K}\mathbf{u} \tag{2}$$

- $\mathbf{f}$: external forces
- $\mathbf{u}$: displacements
- $\mathbf{K}$: global stiffness matrix

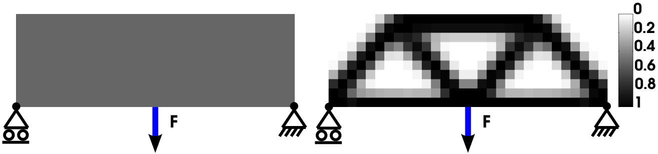
Topology optimization: problem definition



- the domain is discretized in elements
- a design variable x_e is assigned to each element

$$x_e = \begin{cases} 0, & \text{white element} \Rightarrow \text{absence of material} \\ 1, & \text{black element} \Rightarrow \text{presence of material} \end{cases} \quad (3)$$

Topology optimization: problem definition



- for each element e, stiffness E_e is assigned

$$E_e = \begin{cases} E_e^0, & x_e = 1 \\ 0, & x_e = 0 \end{cases} \quad (4)$$

$$E_e = x_e E_e^0 \quad (5)$$

Topology optimization: problem relaxation

SIMP

Solid Isotropic Material with Penalization

- convert descrite problem to continuous

$$x_e = \begin{cases} 0 \\ 1 \end{cases} \Rightarrow 0 \leq x_e \leq 1 \tag{6}$$

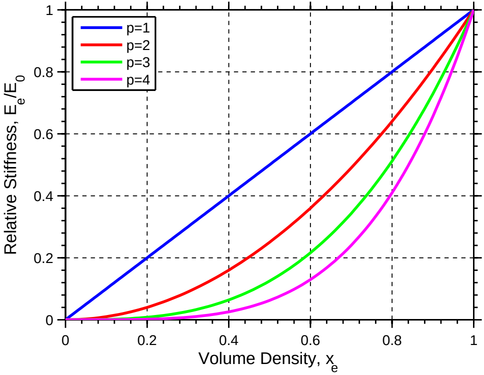
- x_e can is penalized, powered to $p \geq 3$
- move x_e to the limits of space $[0, 1]$

$$E_e = x_e^p E_e^0 \Rightarrow \frac{E_e}{E_e^0} = x_e^p \tag{7}$$

Topology optimization: problem relaxation

SIMP

Solid Isotropic Material with Penalization



Topology optimization: problem definition

- for each element e, stiffness matrix K_e is defined as function of E_e
- 2d plane stress: $\mathrm{K}_e^0 = \frac{E_e^0}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$

$$\mathrm{K}_e = \begin{cases} \mathrm{K}_e^0, & x_e = 1 \\ 0, & x_e = 0 \end{cases} \Rightarrow \mathrm{K}_e = x_e^p \mathrm{K}_e^0 \tag{8}$$

$$\mathrm{K} = \sum_{e=1}^{N} \mathrm{K}_e = \sum_{e=1}^{N} x_e^p \mathrm{K}_e^0, \quad e = 1, \dots, N \tag{9}$$

$$0 < x_{min} \leq x_e \leq 1 \tag{10}$$

Topology optimization: problem definition

- Volume constraint: The optimum structure Ω^{mat} is a fraction of design domain Ω
- V : the volume of Ω^{mat}
- V_0 : the volume of Ω

$$\begin{gathered} \frac{V}{V_0} \leq \phi \Leftrightarrow V \leq \phi V_0 \\ V = \sum_{e=1}^{N} v_e x_e \leq \phi V_0, \end{gathered} \tag{11}$$

- v_e is the volume of element e

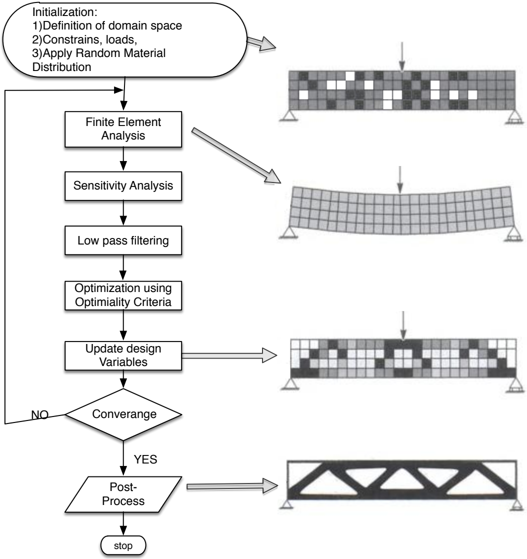
Topology optimization for Structures: Mathematical formulation

$$\min_{\mathbf{u},x_e} c(x_e) = \mathbf{f}^T \mathbf{u},$$

subject to:

$$\begin{aligned} &\mathbf{K}\mathbf{u} = \mathbf{f}, \\ &\sum_{e=1}^{N} v_e x_e \leq \phi V_0, \\ &0 < x_{min} \leq x_e \leq 1, \quad e = 1, \dots, N, \\ &\mathbf{K} = \left(\sum_{e=1}^{N} x_e^p \mathbf{K}_0 \right), \quad p \geq 3 \end{aligned} \tag{12}$$

Topology optimization: the algorithm



Topology optimization: numerical problems

- conditions:
  - type and number of finite elements used
  - optimization algorithm and it's tuning coefficients
  - initial state that topology optimization start its iterative process.
- problems:
  - checkerboard effects
  - mesh dependency
  - multiplicity of solutions

Topology optimization: numerical problems

- checkerboard effects

Topology optimization: numerical problems

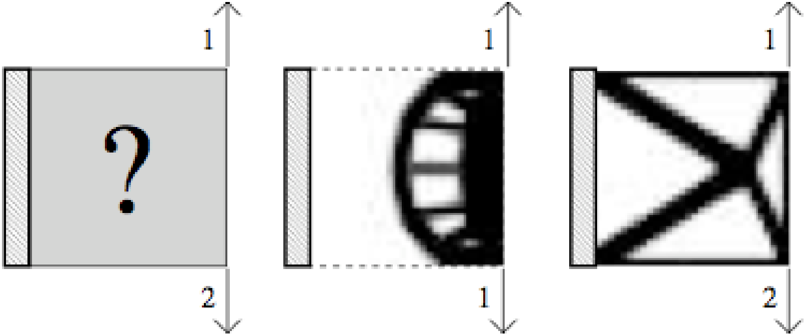
- mesh dependency

(a) Mesh 30×10, 300 elements

(b) Mesh 60×20, 1200 elements

(c) Mesh 120×40, 4800 elements

Topology optimization extensions: Structures multiple load cases



Topology optimization extensions: Structures multiple load cases

- problem expression: minimize compliance for all load cases

$$\begin{cases} \min\limits_{\mathbf{u}_1, x_e} c_1(x_e) & = \mathbf{f}_1^T \mathbf{u}_1, \\ \min\limits_{\mathbf{u}_2, x_e} c_2(x_e) & = \mathbf{f}_2^T \mathbf{u}_2, \\ & \vdots \\ \min\limits_{\mathbf{u}_m, x_e} c_m(x_e) & = \mathbf{f}_m^T \mathbf{u}_m, \end{cases} \tag{13}$$

- m: total number of load cases

Topology optimization extensions: Structures multiple load cases

- problem formulation

$$\min \sum_{i=1}^{m} w_i c_i = \sum_{i=1}^{m} w_i \mathrm{u}_i^T \mathrm{K} \mathrm{u}_i$$

subject to:

$$\begin{aligned} &\left(\sum_{e=1}^{N} x_e^p \mathrm{K}_0\right)\mathrm{u}_i = \mathrm{f}_i, \quad i = 1, \dots, m \\ &\sum_{e=1}^{N} v_e x_e \leq \phi V_0, \\ &0 < x_{min} \leq x_e \leq 1, \quad e = 1, \dots, N, \\ &\sum_{i=1}^{m} w_i = 1 \end{aligned} \tag{14}$$

- w_i: weight for each load case, $i = 1, \dots, m$

Topology optimization extensions: compliant mechanisms

Compliant mechanism is a structure:

- single piece body - jointless
- transforms input loads into motion to another point of the structure, through body deformation
- flexible enough to deliver motion
- stiff enough to bear with the input loading

Compliant mechanisms: advantages

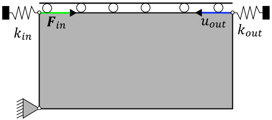
- no joint friction, no backlash, no lubrication
- can be combined with modern actuators (piezoelectric, electromechanical)
- scalable: work in micro, meso, & macro scale
- wide range of applicable materials: aluminum, titanium, steel, ABS, composites, etc.

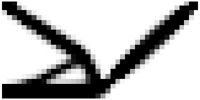
Compliant mechanisms: Applications

Applications of compliant mechanisms:

- MEMS: Micro Electro Mechanical Systems
  - accelerometer sensor
  - pressure sensor
  - gyroscopes
- surgical tools
- aerodynamics: airfoil morphing

Compliant mechanisms: examples





Topology optimization extensions: compliant mechanisms

$$\max_{\mathrm{u},x_e} u_{out} = \mathbf{1}^T \mathrm{u}$$

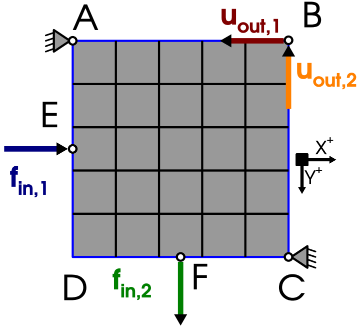
s.t:

$$\begin{aligned}
&\mathrm{Ku} = \mathrm{f} \\
&\sum_{e=1}^{N} v_e x_e \leq \phi V_0 \\
&\mathrm{K} = \left(\sum_{e=1}^{N} x_e^p \mathrm{K}_0\right), \quad p \geq 3 \\
&0 < x_{min} \leq x_e \leq 1, \quad e = 1, \ldots, N
\end{aligned} \tag{15}$$

$$\mathbf{1}^T = [0, 0, 0, \ldots 1 \ldots 0, 0, 0]$$

Topology optimization extensions: multifunctional compliant mechanisms

- are compliant mechanisms that deliver different different motions according to each load case



(a)

Topology optimization extensions: multifunctional compliant mechanisms

- it is a multicriteria problem

$$\begin{cases} \max\limits_{u_1, x_e} c_1(x_e) & = u_{out,1} \\ \max\limits_{u_2, x_e} c_2(x_e) & = u_{out,2}, \\ & \vdots \\ \max\limits_{u_m, x_e} c_m(x_e) & = u_{out,m} \end{cases} \tag{16}$$

- $u_{out,i}, \quad i = 1, \dots, m$ are the separate displacements for each load case

Topology optimization extensions: multifunctional compliant mechanisms

- mathematical formulation

$$\max \sum_{i=1}^{m} w_i c_i = u_{out}^{(\Sigma)} = \sum_{i=1}^{m} w_i \mathrm{1}_i^T \mathrm{u}_i$$

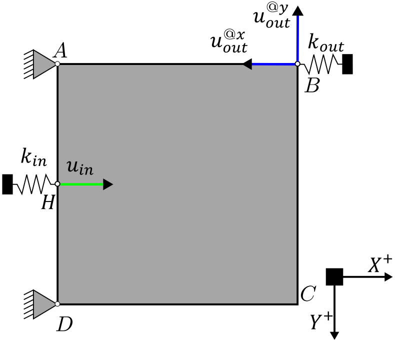
subject to:

$$\begin{aligned} &\mathrm{K}\mathrm{u}_i = \mathrm{f}_i, \quad i = 1, \dots, m \\ &\sum_{e=1}^{N} v_e x_e \leq \phi V_0, \\ &0 < x_{min} \leq x_e \leq 1, \quad e = 1, \dots, N, \\ &\sum_{i=1}^{m} w_i = 1 \end{aligned} \tag{17}$$

Compliant mechanisms: output control

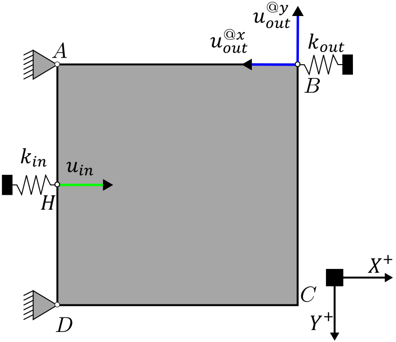
- maximize horizontal displacement: $u_{out}^{@X}$
- minimize vertical displacement: $u_{out}^{@Y}$

$$\begin{cases} \max\limits_{x} \left|u_{out}^{@X}\right| \\ \text{and} \\ \min\limits_{x} \left|u_{out}^{@Y}\right| \end{cases}$$



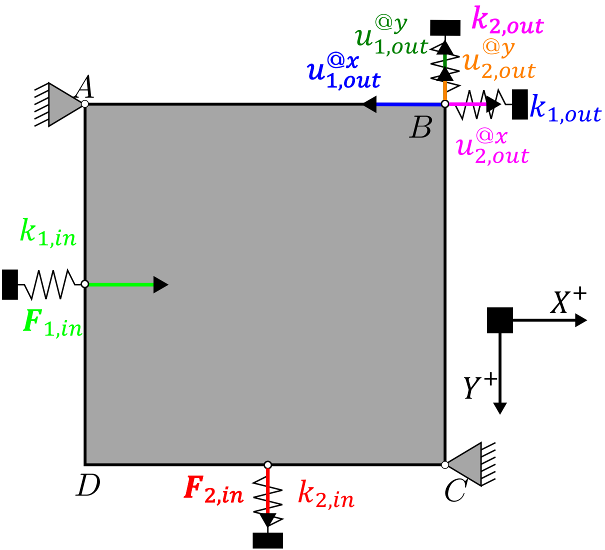
$$\min_{x} \left|u_{out}^{@Y}\right| \Leftrightarrow \max \left|\frac{1}{u_{out}^{@Y}}\right|$$

Compliant mechanisms: output control

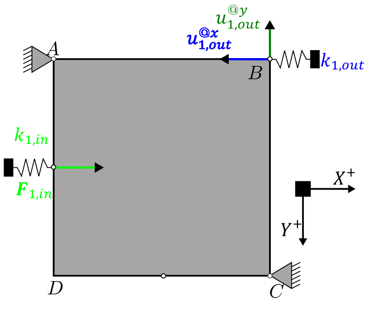


$$\begin{cases} \max\limits_{x} \left| u_{out}^{@X} \right| \\ \text{and} \\ \min\limits_{x} \left| u_{out}^{@Y} \right| \end{cases} \Leftrightarrow \max_{x} \left| \frac{u_{out}^{@X}}{u_{out}^{@Y}} \right| \tag{18}$$

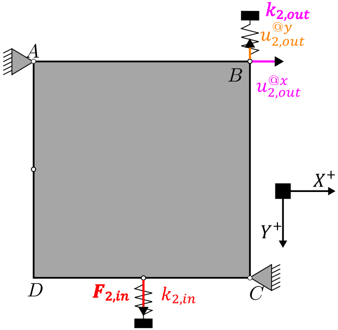
Multifunctional compliant mechanisms: output control



Multifunctional compliant mechanisms: output control



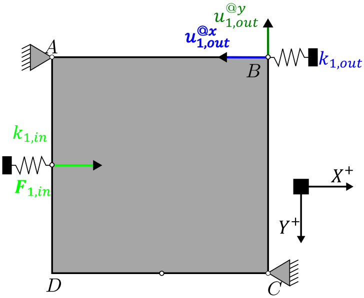
(a) 1st load case



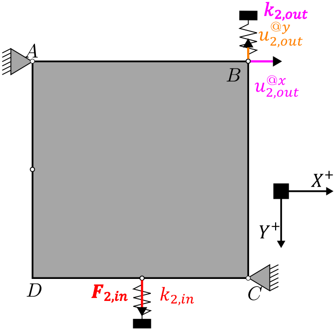
(b) 2nd load case

$$\begin{cases} \max\limits_{x} \left|u_{1,out}^{@X}\right| \\ \min\limits_{x} \left|u_{1,out}^{@Y}\right| \end{cases} \quad \& \quad \begin{cases} \max\limits_{x} \left|u_{2,out}^{@Y}\right| \\ \min\limits_{x} \left|u_{2,out}^{@X}\right| \end{cases} \tag{19}$$

Multifunctional compliant mechanisms: output control



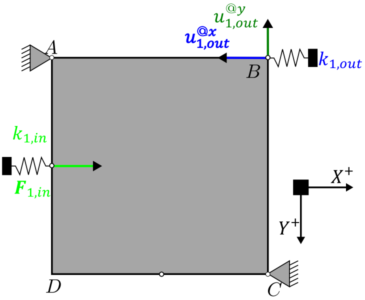
(a) 1st load case



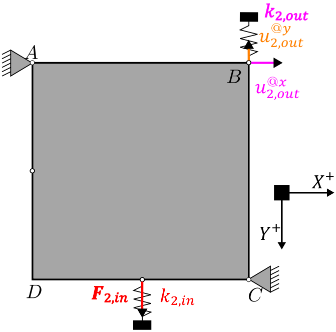
(b) 2nd load case

$$\left\{ \max_{x} \left| \frac{u^{@X}_{1,out}}{u^{@Y}_{1,out}} \right| \quad \& \quad \max_{x} \left| \frac{u^{@Y}_{2,out}}{u^{@X}_{2,out}} \right| \right. \tag{20}$$

Multifunctional compliant mechanisms: output control



(a) 1st load case



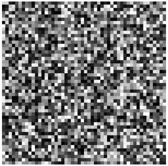
(b) 2nd load case

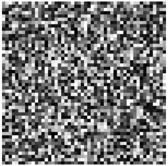
$$\max_{x}\left\{\min\left\{\left|\frac{u_{1,out}^{@X}}{u_{1,out}^{@Y}}\right|,\left|\frac{u_{2,out}^{@Y}}{u_{2,out}^{@X}}\right|\right\}\right\} \tag{21}$$

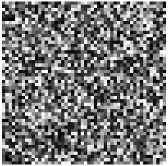
Nonconvex problem-Local Minima

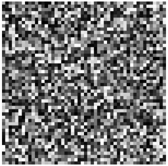
- Structural optimization problems are nonconvex problems
- Topology optimization used for structural optmization problems, depends on the starting point of the procedure
- the appearance of local minima is more often in topology optimization problems for compliant mechanisms
- global optimization is required

Compliant mechanisms: Appearance of local minima


(a)


(b)


(c)


(d)


(e)


(f)


(g)


(h)

The hybrid scheme

Direct use of global optimization is not possible due to

- large number of design variables
- highly nonconvex problem

Direct use of genetic or evolutionary algorithm might be a good idea but:

- tunned up carefully to cope with large # of design variables
- operators like mutation or crossover does not guarantee that the design variable vector represent a structure (no presence of islands of material inside the structure)
- therefore, tailored operators are required in order the structure to have an inversible stiffness matrix

The hybrid scheme

A hybrid scheme is used combining the best features of:

- evolutionary algorithms
  - Differential Evolution
  - Particles Swarm Optimization
- local iterative algorithms

The hybrid scheme-Evolutionary Algorithms

Evolutionary Algorithms features:

- population-based optimization algorithms
- use biological mechanisms:
  - reproduction
  - mutation
  - recombination/crossover
  - selection
- every member of the population is a candidate solution
- every member has a value determined by a fitness function
  - $\max c = \mathbf{1}^T \mathrm{u}$
  - $\max \left| \frac{u_{out}^{@X}}{u_{out}^{@Y}} \right|$, using output control
- evolution of the population is based on a repeated application of the above operators

Differential Evolution

- Introduced by Price & Storn (1995)
- belong to the family of evolutionary algorithms
- Easy to implement, easy parallelization
- Stochastic, population based optimization algorithm
- works with both real, integer & discrete variables
- every member of the population is a candidate solution

Differential Evolution

- every member has a spot (a value) on the search-space of solutions
- based on the evolution of the population in a number of generations
- each individual is moving to a new spot on the search space, based on the relative distance from other individuals
- if the new spot is better that the old, it is accepted, otherwise rejected
- the process is repeated, hoping to find (it is not guaranteed) the best solution in a number of generations

Particles Swarm Optimization

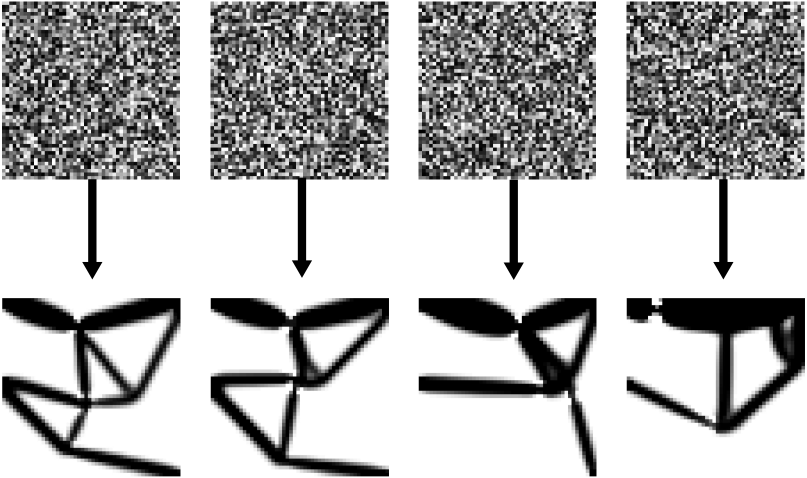
- Introduced by Kennedy & Eberhart (1995)
- simulates the social behaviour of birds, mammals & fish when search for food, in order to find the best solution of a optimization problem
- has all the nice features from DE (easy implementation, parallelization

Particles Swarm Optimization

- each candidate solution is a particle that searches for the optimum
- the swarm of canditate solutions travels on the source space in time increments (like generations)
- each particle is moving, hence has a velocity and a position (a value)
- each particle knows it's previous position and it's best position (personal best)
- the new position of the each particle is effected by its personal best position as well as by the best known positions of the other particles
- the other particles best positions are updates as the swarms moves in time
- it is expected that the swarm will move eventually to the best spot (food)

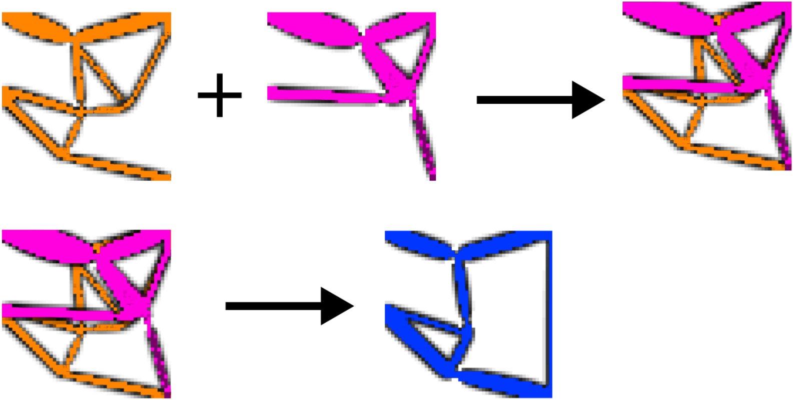
The hybrid scheme

Initialization:

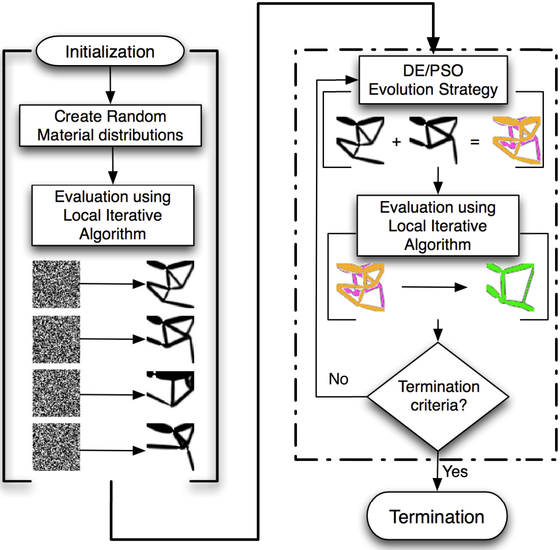


The hybrid scheme

Evolution & Evaluation



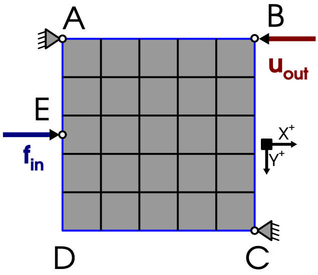
The hybrid scheme: the flow chart



Compliant mechanisms: 1 Load case-w/out output control - DE

- Topology Optimization parameters

Parameter	Value
Discritization	50x50
Design Variables	2500
Degrees of Freedom	5202
Local search iterations	100
SIMP penalty: p	3
Filter radius: r	2
Volume Limit	30%

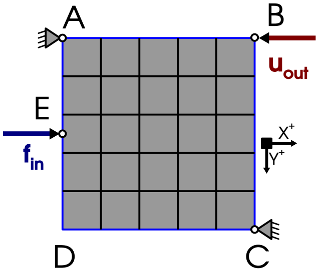


Compliant mechanisms: 1 Load case-w/out output control - DE

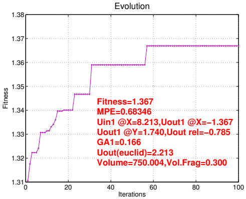
- DE parameters

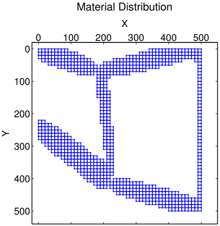
Parameter	Value
Population size	32
Generations	100
Crossover C_r	0.9
Mutation β	1.5
Design Variables	2500

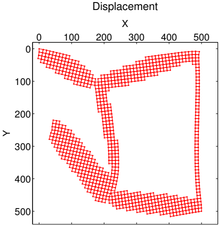
Table – Differential Evolution configuration parameters

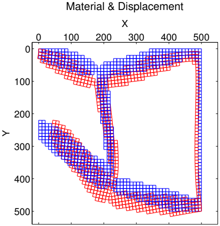


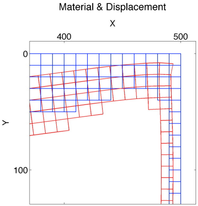
Compliant mechanisms: 1 Load case-w/out output control-DE











Compliant mechanisms: 1 Load case-w/out output control-DE

- Evolution: steps 1 to 8:

(a) step 1

(b) step 2

(c) step 3

(d) step 4

(e) step 5

(f) step 6

(g) step 7

(h) step 8

Compliant mechanisms: 1 Load case-w/out output control-DE

- Evolution: Steps 9 to 15


(a) step 9


(b) step 10


(c) step 11


(d) step 12


(e) step 13

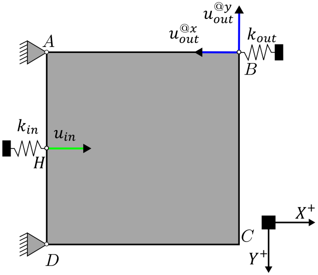

(f) step 14


(g) step 15

Compliant mechanisms: 1 Load case-with output control - PSO

- Topology Optimization parameters

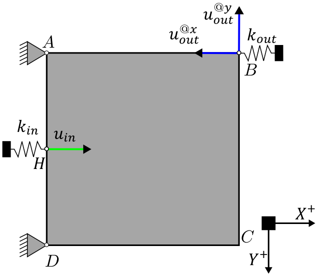
Parameter	Value
Discritization	30x30
Design variables	900
DOFS	1922
Local search iterations	60
SIMP penalty, p	3
Filter Radius, r	1.5
Volume fraction	30%



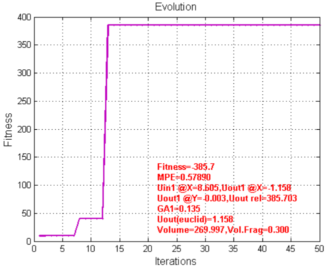
Compliant mechanisms: 1 Load case-with output control - PSO

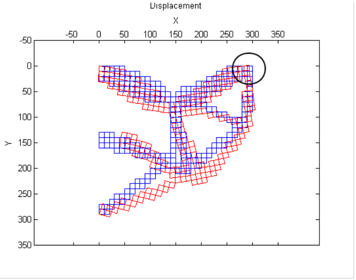
- PSO configuration parameters

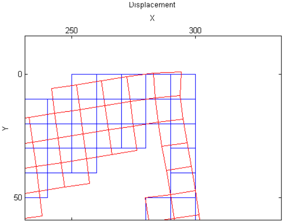
Parameter	Value
Swarm size	20
# PSO local searches	50
Acceleration $c_1 = c_2$	2
Inertia w_{max} & w_{min}	0.9, 0.1
Design variables	900



Compliant mechanisms: 1 Load case-with output control-PSO



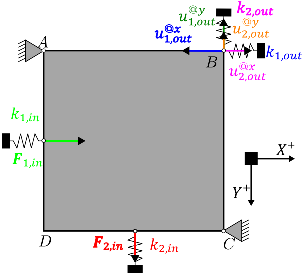




Compliant mechanisms: 2 Loads case-with output control

- Topology Optimization parameters

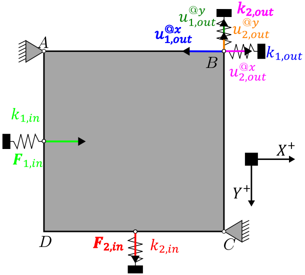
Parameter	Value
Discritization	50x50
Design variables	2500
DOFS	5202
Local search iterations	80
SIMP penalty, p	3
Filter Radius, r	2
Volume fraction	30%



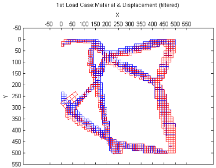
Compliant mechanisms: 2 Loads case-with output control - PSO

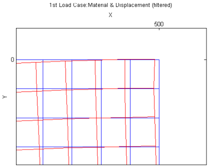
- PSO configuration parameters

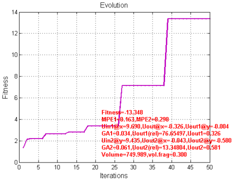
Parameter	Value
Swarm size	20
# PSO local searches	50
Acceleration $c_1 = c_2$	2
Inertia w_{max} & w_{min}	0.9, 0.1
Design variables	2500

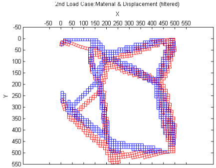


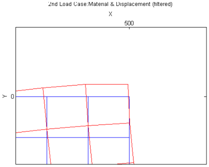
Compliant mechanisms: 2 Loads case-with output control-PSO











Computational Data

- the MATLAB programming enviroment was used
- specialized computational procedures based on MATLAB sparsity features was used for: the assembly of the stiffness matrix as well as for the solving
- Parallelization through MATLAB was used taking advantage the PARFOR command
- MATLAB vectorization features was used for faster implementation of DE & PSO codes

For a typical problem with two load cases and PSO

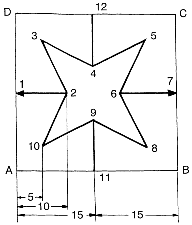
- 50x50 mesh
- swarm size: 20
- # of PSO iteration: 50
- 8 parallel cores used
- time: 6.8 hours

Auxetics materials: Definition

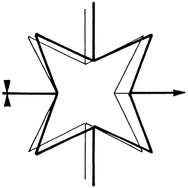
Auxetic materials features:

- the word auxetic comes from the greek word ”αυξητός”
- artificial microstructures with properties that may not be found in nature
- when streched, it become thicker, perpendicular to the applied force
- this occures due to the specific shape of the micro structure
- it can represented by an array of repeated microstructures
- the auxetic material is described by the Negative Poisson’s Ratio (NPR)
- the microstructure can be modeled as an compliant mechanism

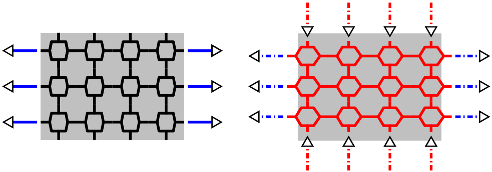
Auxetics materials: Definition



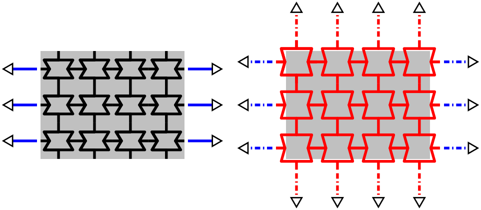
(a)



(b)

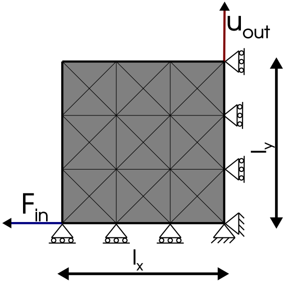


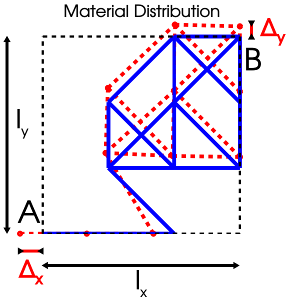
(a)



(b)

Negative Poisson's Ratio: Definition

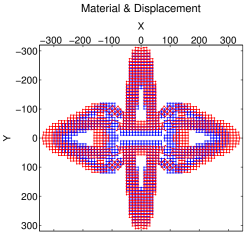


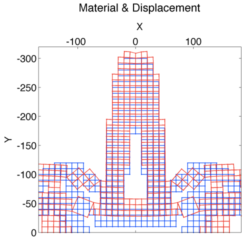


$$\left.\begin{aligned} \nu = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\frac{\Delta y}{l_y}}{\frac{\Delta x}{l_x}} = -\frac{\Delta y}{\Delta x} \\ \Delta x, \Delta y > 0 \end{aligned}\right\} \Rightarrow \nu < 0 \tag{22}$$

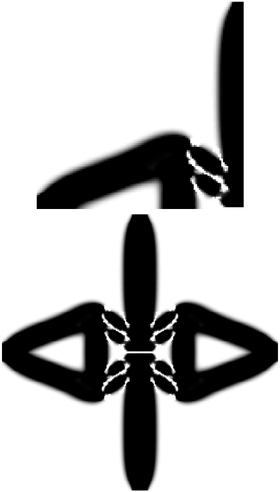
Auxetic materials: mesh 30x30, volume fraction 30%, DE

- Negative Poisson's ratio $\nu = -0.223$

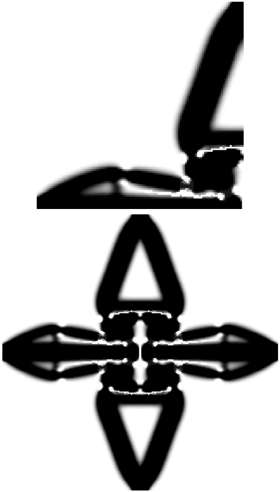




Auxetic materials: mesh 120x120, volume fraction 30%, DE

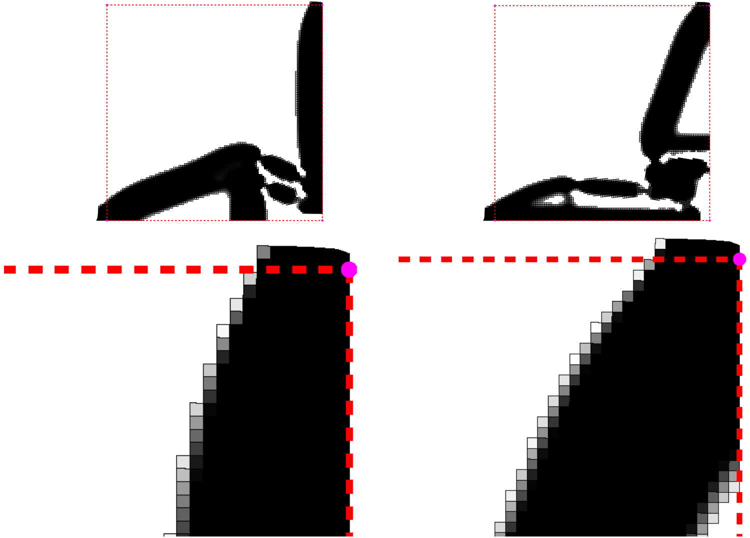


(a) case 1: $\nu = -0.216$

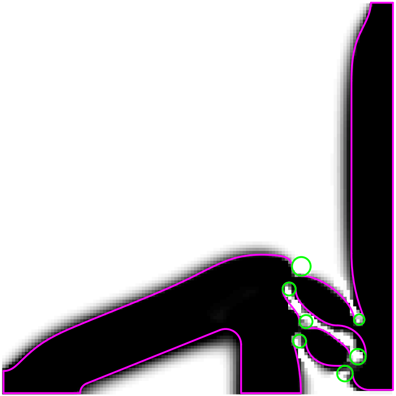


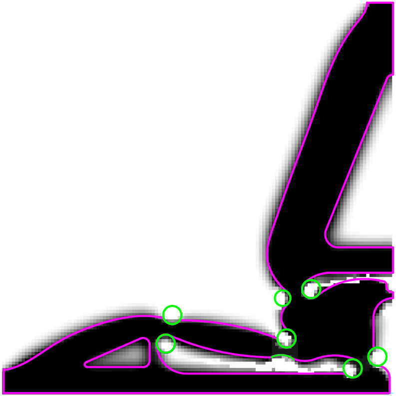
(b) case 2: $\nu = -0.207$

Auxetic materials: mesh 120x120, volume fraction 30%



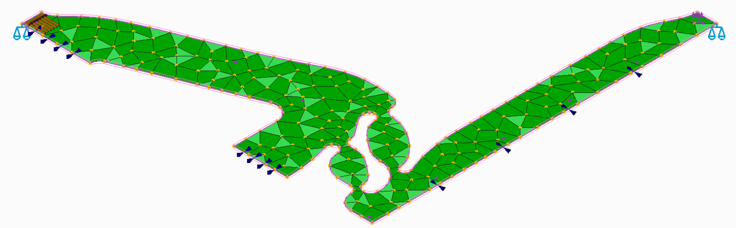
Auxetic materials: mesh 120x120, volume fraction 30%


(a) case 1

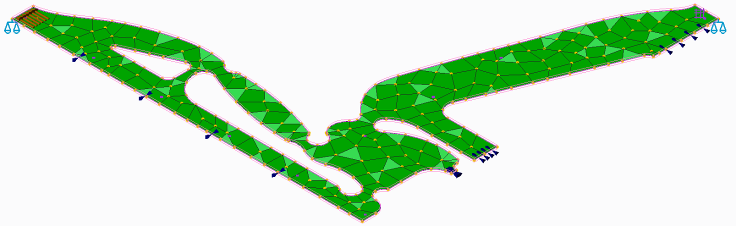

(b) case 2

- stress constraints
- fatique constraints
- robust design

Auxetic materials: mesh 120x120, volume fraction 30%

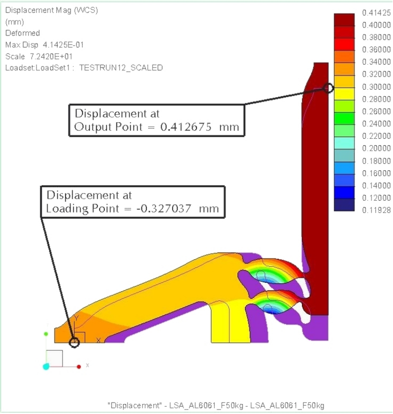


(a) case 1

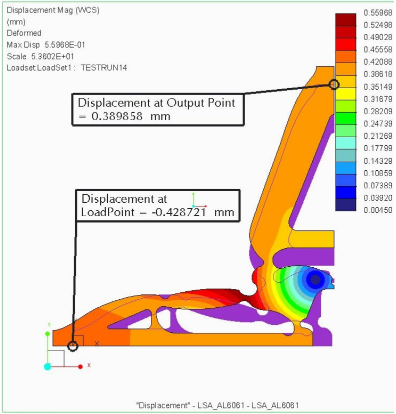


(b) case 2

Auxetic materials: mesh 120x120, volume fraction 30%

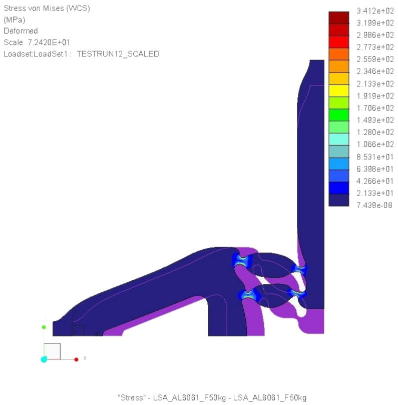


(a) case 1: $\nu = -1.263$

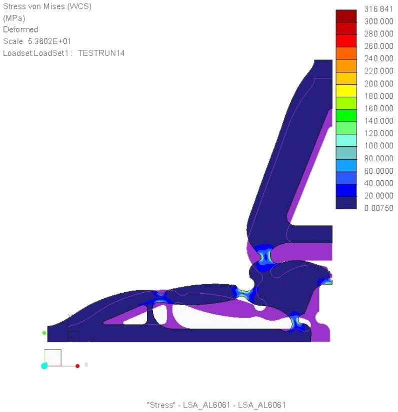


(b) case 1: $\nu = -0.909$

Auxetic materials: mesh 120x120, volume fraction 30%



(a) case 1



(b) case 2

Conclusions

- topology optimization does not guarantee that in complex optimization problems, the resulted material distribution is the optimum.
- the hybrid scheme can overcome this problem, by combining the best features from evolutionary algorithms and iterrative local processes
- relatively can provide better solutions close to the global optimum
- the hybrid scheme can be used as conceptual design tool for any design application

Future Work

Coupling Topology Optimization with:

- geometric nonlinearties for the design of compliant mechanisms
- with contact mechanics
- using elastoplastic materials

Future Work

Coupling Topology Optimization with:

- use of other evolutionary algorithms except of DE & PSO
- use of alternatives to the SIMP: ESO, BESO, level set method
- multiobjective versions of DE or PSO
- parametric investigation of DE and PSO tunning parameters

Published papers

- Kaminakis, N., Stavroulakis, G.E. (2012). ”Topology optimization for compliant mechanisms, using evolutionary algorithms and application on the design of auxetic materials.” JCOMB Composites Part B: Engineering (Elsevier), vol. 43(6), pp. 2655-2668.
- Kaminakis, N., Stavroulakis, G.E. (2012). ”Design of auxetic microstructures using topology optimization.” Structural Longevity, vol. 8(1), pp. 1-6.
- Kaminakis N., Drosopoulos G.A. and Stavroulakis G.E. (2014). ”Design and verification of auxetic microstructures using topology optimization and homogenization.” Archive of Applied Mechanics. September 2015, Volume 85, Issue 9, pp 1289-1306
- G.A. Drosopoulos, N. Kaminakis, N. Papadogianni and G.E. Stavroulakis (2015). ”Mechanical behaviour of auxetic microstructures using contact mechanics and elastoplasticity.” Key Engineering Materials. (accepted for publication)

Πρόγραμμα ΕΣΠΑ 2007-2003
ΗΡΑΚΛΕΙΤΟΣ ΙΙ

Heracleitus II: program co-financed by the European Social Fund (ESF) and Greek national funds, through the operational program "Education and Lifelong Learning", within the National Strategic Reference Framework (NSRF)

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Ευρωπαϊκό Κοινωνικό Ταμείο

ΕΠΙΧΕΙΡΗΣΙΑΚΟ ΠΡΟΓΡΑΜΜΑ
ΕΚΠΑΙΔΕΥΣΗ ΚΑΙ ΔΙΑ ΒΙΟΥ ΜΑΘΗΣΗ
επένδυση στην κοινωνία της γνώσης
ΥΠΟΥΡΓΕΙΟ ΠΑΙΔΕΙΑΣ & ΘΡΗΣΚΕΥΜΑΤΩΝ, ΠΟΛΙΤΙΣΜΟΥ & ΑΘΛΗΤΙΣΜΟΥ
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ΕΣΠΑ
2007-2013
πρόγραμμα για την ανάπτυξη
ΕΥΡΩΠΑΪΚΟ ΚΟΙΝΩΝΙΚΟ ΤΑΜΕΙΟ

Με τη συγχρηματοδότηση της Ελλάδας και της Ευρωπαϊκής Ένωσης

NSRF
2007-2013
programme for development

The End