Joint Resource Allocation and Routing in Wireless Networks via Convex Approximation Techniques

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Outline

1. Convex Approximation Algorithms for Back-Pressure Power Control
   - Back-pressure power control problem (**BPPC**)
   - NP-hardness
   - Successive convex approximation strategy
   - Custom adaptive algorithms
   - Simulation results
   - Concluding remarks
2. Distributed Back-Pressure Power Control for Wireless Multi-hop Networks
   - Distributed implementation of core step of S.A. to BPPC
   - Distributed S.A. Algorithms for BPPC
   - Distributed WMMSE-based algorithms for BPPC
   - Adaptive distributed solutions
   - Simulation results
   - Concluding remarks

Shortest path vs. dynamic back-pressure

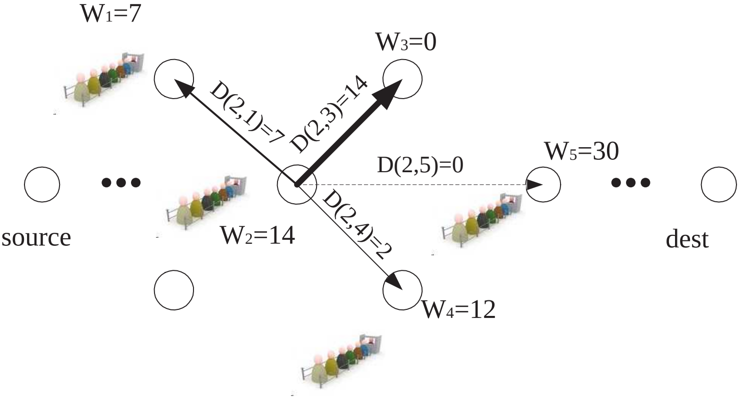
SP

- DP: BF, FW, ...
- Distributed ✓
- Must know arrival rate
- Quasi-static, very slow to adapt to
  - changing arrivals/load
  - availability/failure
  - fading/interference patterns
- Claim: Low delay (shortest path)
- ... but only at low system loads

BP [Tassiulas '92]

- One-hop differential backlog
- Distributed ✓ Lightweight ✓
- Auto-adapts ✓
- Highly dynamic, agile ✓
- Claim: maximal stable throughput (all paths)
- ... but delay can be large - U(load), $\emptyset \rightarrow$ rand walk!

Back-pressure routing



- Favors links with low back-pressure (hence name)
- Backtracking / looping possible!
- Local communication, trivial computation

Back-pressure routing

- Multiple destinations, commodities?
  - multiple queues per node
  - (max diff backlog) winner-takes-all per link
- Wireline: local communication, trivial computation
- Wireless?
- Broadcast medium: interference
- Link rates depend on transmission scheduling, power of other links
- Through appropriate scheduling, power control ...
- obtain ‘favorable’ topology for throughput maximization

Back-pressure power control

SINR

$$\gamma_\ell = \frac{G_{\ell\ell} p_\ell}{\sum_{k \in \mathcal{L}, k \neq \ell} G_{k\ell} p_k + V_\ell}$$

Link capacity

$$c_\ell = \log(1 + \gamma_\ell)$$

BPPC

$$\max_{\{p_\ell\}_{\ell \in \mathcal{L}}} \sum_{\ell \in \mathcal{L}} D_\ell(t) c_\ell$$

$$\text{s.t.} \quad 0 \leq \sum_{\ell : \mathrm{Tx}(\ell) = i} p_\ell \leq P_i, \forall i \in \mathcal{N}$$

$$0 \leq p_\ell \leq P_\ell, \ell \in \mathcal{L}$$

Diff backlog link $\ell = (i \to j)$ @ time t

$$D_\ell(t) := \begin{cases} \max\{0, W_i(t) - W_j(t)\}, & j \neq N \\ W_i(t), & j = N. \end{cases}$$

$\mathcal{F}$ flows: $D_\ell(t) := \max_{f \in \mathcal{F}} D_\ell^{(f)}(t)$

Back-pressure power control

BPPC

$$\max_{\{p_\ell\}_{\ell\in\mathcal{L}}} \sum_{\ell\in\mathcal{L}} D_\ell(t) c_\ell$$

$$\text{s.t.} \quad 0 \le \sum_{\ell:\mathrm{Tx}(\ell)=i} p_\ell \le P_i, \forall i \in \mathcal{N}$$

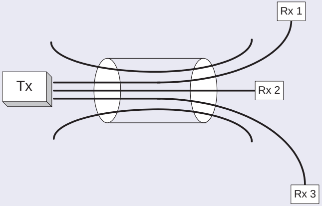
$$0 \le p_\ell \le P_\ell, \ell \in \mathcal{L}$$

[Tassiulas *et al*, '92 →]

- Max stable throughput ✓
- Backbone behind modern NUM
- Core problem in wireless networking
- complexity?

Reminiscent of ...

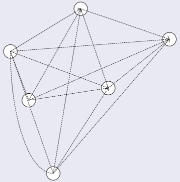
DSL: sum-rate maximization



Single-hop DSL

- Listen-while-talk ✓
- Dedicated (Tx,Rx)
- Free choice of $G_{k,\ell}$'s
- NP-hard [Luo, Zhang]

BPPC

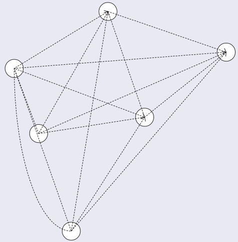


Multi-hop network

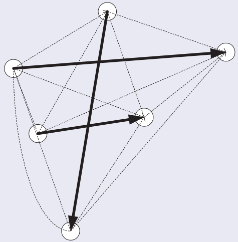
- No listen-while-talk X
- Shared Tx, Rx $\Rightarrow$
- Restricted $G_{k,\ell}$'s, e.g.: k, ℓ depart from $i \rightarrow G_{k,\ell} = G_{\ell,\ell}$, $G_{\ell,k} = G_{k,k}$
- NP-hard?

Peel off

Generic backlogs



Choosing backlogs



- Backlog reduction → BPPC contains DSL → also NP-hard

DSL $\rightarrow$ Multi-hop network optimization

- Can reuse tools from DSL
- In particular, lower approximation algorithms:
  - High SINR $\rightarrow$ ($\gamma_\ell \gg 1$)$\rightarrow$ $\log(\gamma_\ell) \approx \log(1+\gamma_\ell)$ $\rightarrow$ Geometric Programming
  - Successive approximation from below: SCALE [Papandriopoulos and Evans, 2006]
  - Uses

$$\alpha \log(z) + \beta \leq \log(1+z) \text{ for } \begin{cases} \alpha = \frac{z_o}{1+z_o} \\ \beta = \log(1+z_o) - \frac{z_o}{1+z_o}\log(z_o) \end{cases}$$

    tight at z_o; $\rightarrow \log(z) \leq \log(1+z)$ as $z_o \rightarrow \infty$
  - Start from high SINR ($\alpha = 1, \beta = 0$), tighten bound at interim solution
  - *Successive Convex Approximation Approach*:
    - $\mathbf{p}(t) \rightarrow$ new $\alpha_\ell(t)$, $\beta_\ell(t)$ at $z_o = \gamma_\ell(\mathbf{p}(t)), \forall \ell \in \mathcal{L} \rightarrow$ new $\mathbf{p}(t)$, $\forall t$
  - Majorization

Key difference with DSL

- BPPC problem must be solved repeatedly for every slot
- Batch algorithms: prohibitive complexity
- Need adaptive, lightweight solutions (to the extent possible)
- Built custom interior point algorithms
- Normally, one would init using solution of previous slot; take refinement step
- Doesn't work ...
- Why?

Proper warm-start

- No listen-while-talk, shared Tx/Rx
- Push-pull 'wave' propagation
- Solution from previous slot very different from one for present slot
- Even going back a few slots
- Fixed/slowly varying ph. l. propagation conditions
- Deterministic fixed-rate (random but bounded) arrivals
- Quasi-periodic behavior emerges (stable setups) - return to one already visited state
- Idea: hold record of solutions for W previous slots. $W >$ upper bound on period
- W evaluations of present objective function (cheap!)
- Pick the best to warm-start present slot
- Needs few IP steps to converge

Adaptive Successive Convex Approximation

1. $\forall\ t \rightarrow \{D_\ell(t)\}_{\ell \in \ell}$
2. For $t = 1 \rightarrow$ random $\tilde{\mathbf{p}}_o(t)$ satisfying log-power constraints;
   else for $t \in [2,\ W]$ set:

$$\tilde{\mathbf{p}}_o(t) = \arg \max_{\tilde{\mathbf{p}} \in \{\tilde{\mathbf{p}}(1),\ldots,\tilde{\mathbf{p}}(t-1)\}} f(\tilde{\mathbf{p}})$$

$$f(\tilde{\mathbf{p}}) := \sum_{\ell=1}^{L} D_\ell(t) \left(\alpha_\ell \left(\tilde{G}_{\ell\ell} + \tilde{p}_\ell - \log \left(\sum_{\substack{k=1 \\ k \neq \ell}}^{L} e^{\tilde{G}_{k\ell} + \tilde{p}_k} + e^{\tilde{V}_\ell} \right) \right) + \beta_\ell \right)$$

   with α_ℓ, β_ℓ, $\forall \ell \in \mathcal{L}$, updated $\forall\ \tilde{\boldsymbol{p}} \in \{\tilde{\mathbf{p}}(1), \ldots, \tilde{\mathbf{p}}(t-1)\}$,
   else $(t \geq\ W + 1)$ set:

$$\tilde{\mathbf{p}}_o(t) = \arg \max_{\tilde{\mathbf{p}} \in \{\tilde{\mathbf{p}}(t-\ W),\ldots,\tilde{\mathbf{p}}(t-1)\}} f(\tilde{\mathbf{p}})$$

3. Update $\alpha_\ell(t)$, $\beta_\ell(t)$ for $\tilde{\boldsymbol{p}}_o$, i.e., at $z_o = \gamma_\ell(\mathbf{p}_o(t))$, $\forall \ell \in \mathcal{L}$
4. **repeat**: Solve Core problem $\rightarrow$ new $\tilde{\boldsymbol{p}}_o(t) \rightarrow$ new $\alpha_\ell(t)$, $\beta_\ell(t)$ for $\tilde{\boldsymbol{p}}_o(t)$, $\forall \ell \in \mathcal{L} \rightarrow$ new $\tilde{\boldsymbol{p}}_{\mathbf{o}}(t)$ ... **until** convergence

Prior art for BPPC

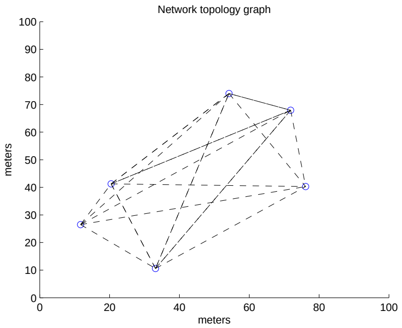
- Low-complexity algorithms for BPPC (multihop CDMA wireless networks) [Giannoulis *et al* 2006]
- high-SINR assumption
- distributed
- run in parallel with system-evolution (real time)

Back Pressure Best Response

$$D_\ell(t) \to \pi_\ell(t) = \frac{D_\ell(t)}{I_\ell(\mathbf{p}(t)) + V_\ell}, \quad \text{where } I_\ell(\mathbf{p}(t) := \frac{1}{G} \sum_{\substack{k=1 \\ k \neq \ell}}^{L} G_{k\ell} p_k(t)$$

$$\pi_\ell(t) \to \forall k \neq \ell \to p_\ell(t+1) = \min\left(\frac{D_\ell(t)}{\sum_{\substack{k=1 \\ k \neq \ell}}^{L} \pi_k(t) \frac{1}{G} G_{\ell k}},\ P_\ell^{max} \right), \forall \ell \in \mathcal{L}, \forall t$$

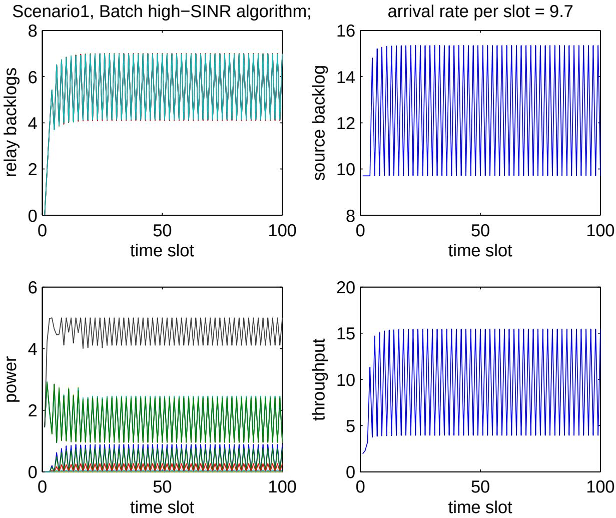
Simulation setup



- $N = 6$ nodes, low-left = s, top-right = d, $L = 25$ links
- $G_{\ell,k} \sim 1/d^4$, d : distance $\mathrm{Tx}(\ell)$, $\mathrm{Rx}(k)$, $G = 128$,
- no-listen-while-talk: if $\mathrm{Rx}(\ell) = \mathrm{Tx}(k) \rightarrow G_{k,\ell} = 1/\mathrm{eps}$
- $V_\ell = 10^{-12}$, $P_\ell = 5$, $\forall \ell$
- Deterministic fixed-rate arrivals

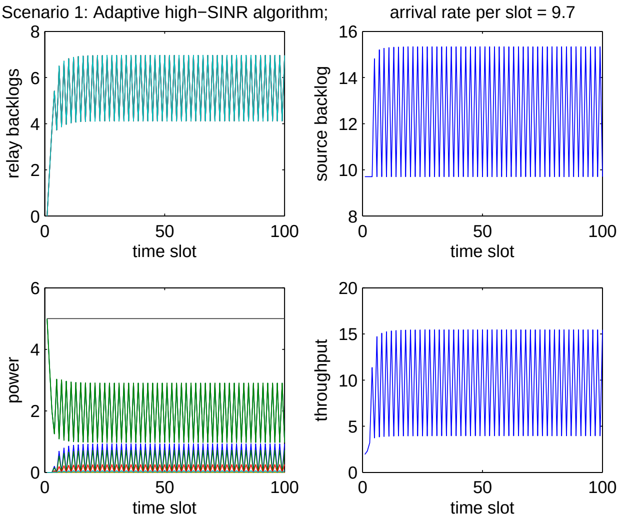
Batch high-SINR

- Maximum stable arrival rate: 9.7 packets per slot (pps)



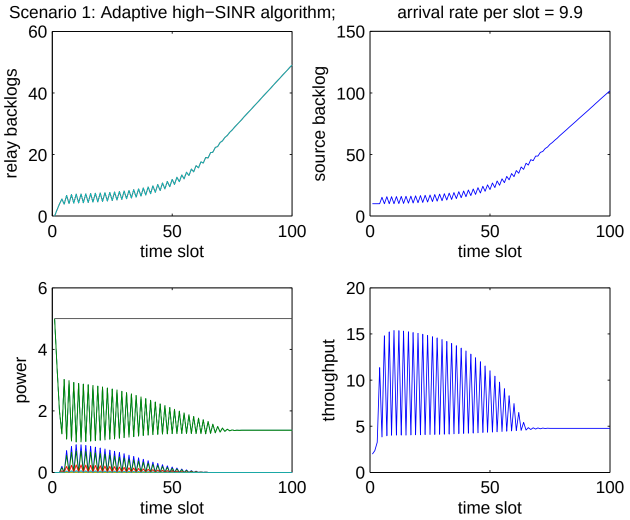
Adaptive high-SINR

- Maximum stable arrival rate: 9.7 pps



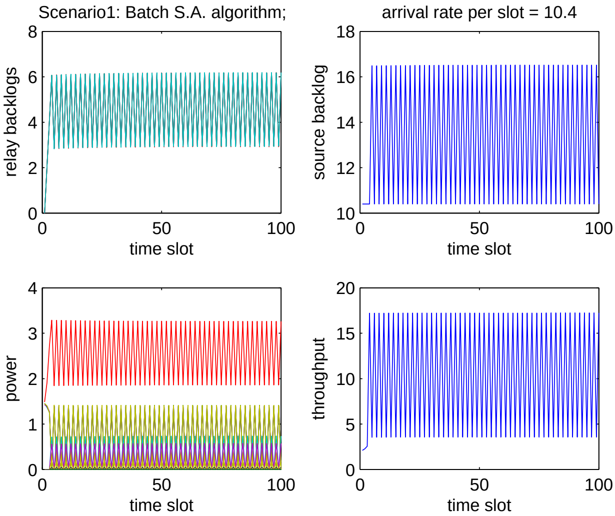
Adaptive high-SINR

- Unstable setup; arrival rate: 9.9 pps



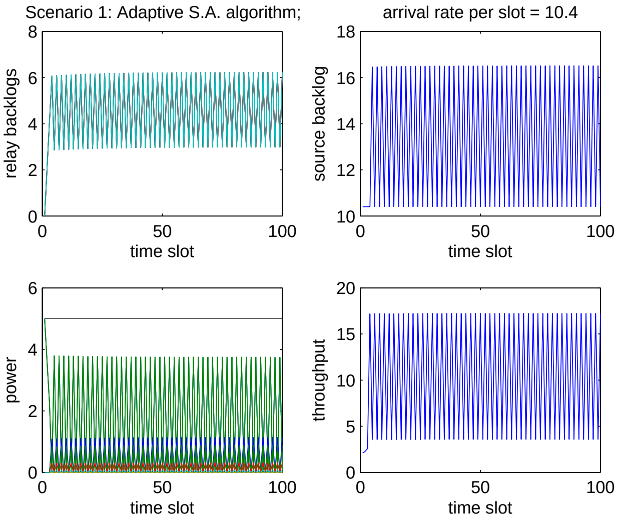
Batch Successive Approximation

- Maximum stable rate: 10.4 pps



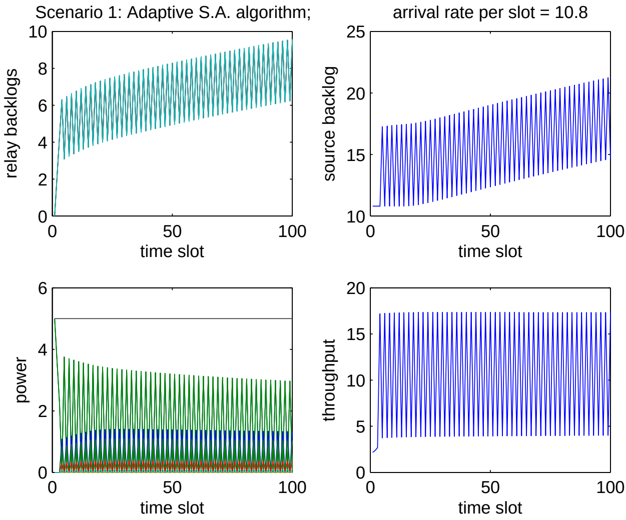
Adaptive Successive Approximation

- Maximum stable rate: 10.4 pps



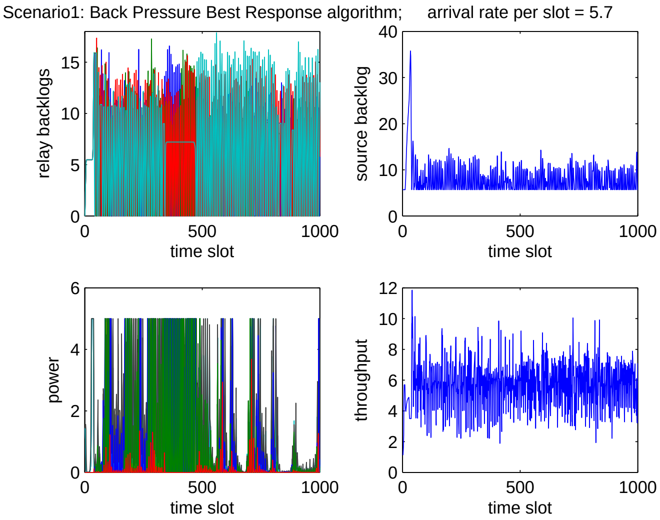
Adaptive Successive Approximation

- Unstable setup; arrival rate: 10.8 pps



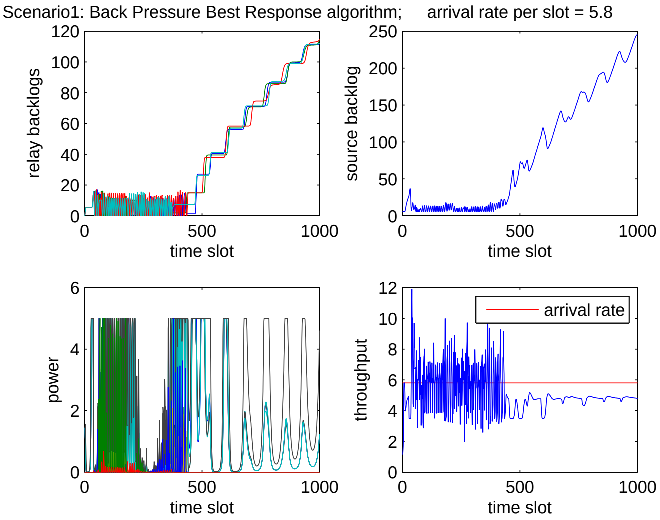
BP Best Response

- Maximum arrival rate: 5.7 pps



BP Best Response

- Unstable setup; arrival rate: 5.8 pps



Throughput & average complexity comparison

Table: Attainable stable arrival rates in packets per slot.

Scenario	B/A high-SINR	B/A successive	Best Resp
Scenario 1	9.7	10.4	5.7
Scenario 2	2.4	7.5	2.1
Scenario 3	12.6	15.7	4.4

BP Best Response

- BP Best Response is very cheap w.r.t. computation
- Indicatively, average run-time: $\approx$ 0.0085 sec per slot
- More than 2 orders faster than adaptive version high-SINR / S.A.

S.A.-BPPC

- Worst case complexity: $\mathcal{O}(L^{3.5})$ for batch / $\mathcal{O}(L^3)$ for adaptive
- Avg. run-time for batch: high-SINR $\approx$ 8 sec / S.A. $\approx$ 20 sec per slot
- Avg. run-time for adaptive: high-SINR / S.A. $\approx$ 1 to 1.5 sec per slot

Concluding remarks

- S.A. approach (DSL literature) is meaningful;
- Manifold improvement in end-to-end throughput vs high-SINR-based alg.
- Much shorter backlogs / queueing delays, much faster transient response compared to BPBR
- Push-pull wave propagation, periodic behavior for stable setups ✓

Distributed BPPC

Core (maximization) step of S.A. approach to BPPC

$$\max_{\{\tilde{p}_\ell \leq \tilde{P}_\ell\}_{\ell \in \mathcal{L}}} \sum_{\ell=1}^{L} D_\ell(t) \underline{c}_\ell$$

$$\underline{c}_\ell := \left(\alpha_\ell(t) \left(\tilde{G}_{\ell\ell} + \tilde{p}_\ell - \log \left(\sum_{\substack{k=1 \\ k \neq \ell}}^{L} e^{\tilde{G}_{k\ell} + \tilde{p}_k} + e^{\tilde{V}_\ell} \right) \right) + \beta_\ell(t) \right)$$

- *Successive Convex Approximation Approach*: $\forall t$,
  - $\mathbf{p}(t) \rightarrow$ new $\alpha_\ell(t)$, $\beta_\ell(t)$ at $z_o = \gamma_\ell(\mathbf{p}(t)), \forall \ell \in \mathcal{L} \rightarrow$ new $\mathbf{p}(t)$
- Good news: manifold improvement in end-to-end throughput relative to the prior art
- Centralized implementation
- Practical power control $\leftrightarrow$ *distributed* across network

Towards distributed implementation of the core step

- **Goal**: *decomposition* into L *parallel* subproblems

Key step: Shift coupling of $\{\tilde{p}_\ell\}_{\ell\in\mathcal{L}}$ from objective to constraints

- auxiliary variables $\{\tilde{i}_{\ell k}\}_{k\neq\ell} :=$ interference to ℓ from $k\neq\ell$, $\forall\ell\in\mathcal{L}$
- add associated constraints: $\tilde{G}_{k\ell}+\tilde{p}_k=\tilde{i}_{\ell k}, \forall k\neq\ell, \forall\ell\in\mathcal{L}$

$$\min_{\tilde{\boldsymbol{p}},\left\{\tilde{\boldsymbol{i}}_\ell\right\}_{\ell\in\mathcal{L}}} \sum_{\ell=1}^{L} -D_\ell\alpha_\ell\left(\tilde{G}_{\ell\ell}+\tilde{p}_\ell\right)+D_\ell\alpha_\ell\log\left(\sum_{\substack{k=1\\k\neq\ell}}^{L} e^{\tilde{i}_{\ell k}}+e^{\tilde{V}_\ell}\right)-D_\ell\beta_\ell$$

$$\text{subject to}\quad \tilde{G}_{k\ell}+\tilde{p}_k=\tilde{i}_{\ell k},\quad \forall k\neq\ell,\quad \forall\ell\in\mathcal{L},$$

$$\text{subject to}\quad \tilde{p}_\ell\leq\tilde{P}_\ell,\quad \ell\in\mathcal{L}$$

- $\tilde{\boldsymbol{i}}_\ell := \left\{\tilde{i}_{\ell k}\right\}_{k\neq\ell}$, $\tilde{\boldsymbol{p}} := \{\tilde{p}_\ell\}_{\ell\in\mathcal{L}}$

The augmented Lagrange function

- Augmented Lagrange function with *penalty parameter* ρ
- $\{\gamma_{\ell k}\}_{k\neq\ell} :=$ Lagrange multipliers $\forall \ell \in \mathcal{L} \mapsto \tilde{G}_{k\ell} + \tilde{p}_k = \tilde{i}_{\ell k}, \forall k \neq \ell$
- $\lambda_\ell :=$ Lagrange multiplier $\mapsto \tilde{p}_\ell \leq \tilde{P}_\ell, \forall \ell \in \mathcal{L}$

Decomposition of augmented Lagrangian

$$L_\rho = \sum_{\ell=1}^{L} \left(-D_\ell \alpha_\ell \left(\tilde{G}_{\ell\ell} + \tilde{p}_\ell\right) + D_\ell \alpha_\ell \log \left(\sum_{\substack{k=1 \\ k\neq\ell}}^{L} e^{\tilde{i}_{\ell k}} + e^{\tilde{V}_\ell} \right) \right.$$
$$-D_\ell \beta_\ell + \lambda_\ell \left(\tilde{p}_\ell - \tilde{P}_\ell\right) + \sum_{\substack{k=1 \\ k\neq\ell}}^{L} \gamma_{k\ell} \tilde{G}_{\ell k} + \tilde{p}_\ell \left(\sum_{\substack{k=1 \\ k\neq\ell}}^{L} \gamma_{k\ell} \right)$$
$$\left. - \sum_{\substack{k=1 \\ k\neq\ell}}^{L} \gamma_{\ell k} \tilde{i}_{\ell k} \right) + \frac{\rho}{2} \sum_{\ell=1}^{L} \sum_{\substack{k=1 \\ k\neq\ell}}^{L} \left(\tilde{G}_{k\ell} + \tilde{p}_k - \tilde{i}_{\ell k}\right)^2$$

Utilizing Alternating Direction Method of Multipliers

Noteworthy points

- $\tilde{p}_\ell$ and $\{\tilde{i}_{\ell k}\}_{k \neq \ell}$ are *local primal* variables for link ℓ
- λ_ℓ and $\{\gamma_{\ell k}\}_{k \neq \ell}$ are *local dual* variables for link ℓ
- $L_\rho = \sum_{\ell=1}^{L} L_\ell \left(\tilde{p}_\ell, \tilde{\boldsymbol{i}}_\ell, \lambda_\ell, \{\gamma_{\ell k}\}_{k \neq \ell} \right)$ + regularization term
- Quadratic regularization term $\rightarrow$ *strict* convexity w.r.t. $\tilde{\boldsymbol{p}}$

- **ADMoM** with dual ascent method $\rightarrow$ decentralized solution possible
- ADMoM $\rightarrow$ favorable convergence properties in our context, unlike dual decomposition method
- ADMoM's steps $\mapsto$ update $\tilde{\boldsymbol{p}}$, $\{\tilde{\boldsymbol{i}}_\ell\}_{\ell \in \mathcal{L}}$, and $\{\gamma_{\ell k}\}_{\ell \in \mathcal{L}, k \neq \ell}$
- Projected gradient step $\mapsto$ update $\boldsymbol{\lambda}$
- L parallel subproblems: $\forall \ell \in \mathcal{L}$ the iterates boil down to ...

ADMoM and dual ascent method

$\tilde{p}_\ell$-optimization step, $\forall \ell \in \mathcal{L}$ ($s :=$ iteration index)

$$\tilde{p}_\ell(s) := \arg\min_{\tilde{p}_\ell} -D_\ell \alpha_\ell \tilde{p}_\ell + \lambda_\ell(s-1)\tilde{p}_\ell + \tilde{p}_\ell \left(\sum_{\substack{k=1 \\ k \neq \ell}}^{L} \gamma_{k\ell}(s-1) \right)$$

$$+ \frac{\rho}{2} \sum_{\substack{k=1 \\ k \neq \ell}}^{L} \left(\tilde{G}_{\ell k} + \tilde{p}_\ell - \tilde{i}_{k\ell}(s-1) \right)^2$$

- convex quadratic in $\tilde{p}_\ell \rightarrow$ closed form update
- Available control channels among nodes $\rightarrow$ information exchanges
- Local link gain information: $\mathrm{Tx}(\ell)$ knows $G_{\ell k}$ to $\mathrm{Rx}(k)$
- **Feedback requirements** from interfering links:
  - dual variables $\{\gamma_{k\ell}(s-1)\}_{k \neq \ell}$
  - auxiliary variables $\left\{\tilde{i}_{k\ell}(s-1)\right\}_{k \neq \ell}$

ADMoM and dual ascent method

$\tilde{i}_\ell$ -optimization step, $\forall \ell \in \mathcal{L}$

$$\left\{\tilde{i}_{\ell k}\right\}_{k \neq \ell}(s) := \arg \min_{\left\{\tilde{i}_{\ell k}\right\}_{k \neq \ell}} D_\ell \alpha_\ell \log\left(\sum_{\substack{k=1 \\ k \neq \ell}}^{L} e^{\tilde{i}_{\ell k}} + e^{\tilde{V}_\ell}\right) - \sum_{\substack{k=1 \\ k \neq \ell}}^{L} \gamma_{\ell k}(s-1)\tilde{i}_{\ell k}$$

$$+\frac{\rho}{2} \sum_{\substack{k=1 \\ k \neq \ell}}^{L} \left(\left(\tilde{G}_{k\ell} + \tilde{p}_k\right)(s) - \tilde{i}_{\ell k}\right)^2$$

- solved e.g., via damped Newton's method
- **Requires**: interference $\left(\tilde{G}_{k\ell} + \tilde{p}_k\right)(s)$ from $\forall k \neq \ell$ to ℓ at iteration $s \rightarrow$ estimated by ℓ, or communicated to ℓ

ADMoM and dual ascent method: optimization steps

$\gamma_{\ell k}$- update step, $\forall k \neq \ell, \ell \in \mathcal{L}$

$$\gamma_{\ell k}(s) := \gamma_{\ell k}(s-1) + \rho\left(\left(\tilde{G}_{k\ell} + \tilde{p}_k\right)(s) - \tilde{i}_{\ell k}(s)\right)$$

- **step-size**: *strictly equal* to ρ, and $\rho > 0$, according to ADMoM

λ_ℓ- projected gradient step, $\forall \ell \in \mathcal{L}$

$$\lambda_\ell(s) := \left[\lambda_\ell(s-1) + \delta_s\left(\tilde{p}_\ell(s) - \tilde{P}_\ell\right)\right]_0^+$$

- **step-size**: e.g., $\delta_s = \delta_1/s,\ \delta_1 > 0$, or a sufficiently small constant $\delta > 0$

Distributed convex approximation of BPPC

Core step 1 algorithm (global constant ρ)

Given D_ℓ, α_ℓ, β_ℓ, $\forall \ell \in \mathcal{L}$, and $s :=$ iteration counter

- **Initialization:** For $s = 0$, set: $\rho > 0$, $\delta_1 > 0$, $\{\lambda_\ell(0)\}_{\ell=1}^{L} > 0$, $\{\gamma_{\ell k}(0)\}_{\ell \in \mathcal{L}, k \neq \ell} > 0$, and $\left\{\tilde{i}_{\ell k}(0)\right\}_{\ell \in \mathcal{L}, k \neq \ell}$ random
- $\forall \ell \in \mathcal{L}$: transmit initial $\gamma_{\ell k}(0)$ and $\tilde{i}_{\ell k}(0)$ to link k, $\forall k \neq \ell$
- **Repeat:** Set $s := s + 1$
  1. $\forall \ell \in \mathcal{L}$: $\tilde{p}_\ell$**-optimization step** to obtain $\tilde{p}_\ell(s)$
  2. $\forall \ell \in \mathcal{L}$: $\tilde{\boldsymbol{i}}_\ell$**-optimization step** to obtain $\tilde{\boldsymbol{i}}_\ell(s)$
  3. $\forall \ell \in \mathcal{L}$: $\gamma_{\ell k}$**-update step** $\forall k \neq \ell$ to obtain $\{\gamma_{\ell k}(s)\}_{k \neq \ell}$
  4. $\forall \ell \in \mathcal{L}$: λ_ℓ**-update step** to obtain $\lambda_\ell(s)$
  5. $\forall \ell \in \mathcal{L}$: *transmit* $\gamma_{\ell k}(s)$ and $\tilde{i}_{\ell k}(s) \rightarrow$ link k, $\forall k \neq \ell$
- **Until:** convergence (within ϵ-accuracy); then $\tilde{p}_\ell^{opt} := \tilde{p}_\ell(s)$, $\forall \ell \in \mathcal{L}$

Distributed core step algorithm

- Convergence *should* be based on *local* computation and communication
- Each link may keep track of a local metric, e.g.,
  - successive differences of its local augmented Lagrange function
  - norm of its residual local equality constraint violation vector $\mathbf{r}_\ell(s)$, with elements $r_{\ell k}(s) := \tilde{G}_{k\ell} + \tilde{p}_k(s) - \tilde{i}_{\ell k}(s)$, $\forall k \neq \ell$
- Local metric under ϵ → local convergence
- ***Termination*** of the distributed protocol:
  - each link maintains *binary flag*: **1** ↔ convergence w.r.t. its local metric (within given ϵ) is *achieved*
  - **distributed consensus-on-the-min** algorithm among links → iterates terminate once *all* links reach convergence
- Variations of ADMoM [Bertsekas '96, Boyd 2011]
  - *adaptive local penalty parameters* → accelerate convergence (see thesis)
  - core step 2: less dependent on initial choice of penalty parameters

Distributed adaptive S.A. algorithm for BPPC

Distributed adaptive S.A. algorithm for BPPC (warm re-start)

- $\forall\ t \rightarrow \{D_\ell(t)\}_{\ell\in\mathcal{L}}$ $\qquad W >$ expected period
- ***Initialization***: For $t = 1$:
  $\alpha_\ell(t) = 1, \beta_\ell(t) = 0, \forall \ell \in \mathcal{L}\ \lambda_{\ell,o} > 0,\ \{\gamma_{\ell k,o}\}_{k\neq\ell} > 0,\ \tilde{\boldsymbol{i}}_{\ell,o}$ random
  For $t \in [2,\ W]$ set $\forall \ell \in \mathcal{L}$:

$$\tau_o = \arg \min_{\tau\in\{t-1,\ldots,1\}} |D_\ell(t) - D_\ell(\tau)|$$

For $t \geq W + 1$ set $\forall \ell \in \mathcal{L}$:

$$\tau_o = \arg \min_{\tau\in\{t-1,\ldots,t-W\}} |D_\ell(t) - D_\ell(\tau)|$$

$$\alpha_\ell(t) := \alpha_\ell^{opt}(t-\tau_o), \quad \beta_\ell(t) := \beta_\ell^{opt}(t-\tau_o)$$

$$\lambda_\ell := \lambda_\ell^{opt}(t-\tau_o), \quad \{\gamma_{\ell k}\}_{k\neq\ell} := \left\{\gamma_{\ell k}^{opt}(t-\tau_o)\right\}_{k\neq\ell}, \quad \tilde{\boldsymbol{i}}_\ell := \tilde{\boldsymbol{i}}_\ell^{opt}(t-\tau_o)$$

Weighted sum-MSE Minimization for WSR Maximization

- Distributed Sum-Utility Maximization for MIMO IBC [Christensen *et al* 2008], [Shi *et al*, 2011]
- In [Shi *et al*, 2011]:
- *Proof*: **WSR Maximization** is ***equivalent*** to properly **weighted sum-MSE Minimization**
- *Proof*: Iterative **WMMSE** $\rightarrow$ **local optimal** (st. point) of **WSRM**
- Use iterative WMMSE to approx. solve BPPC at physical layer
- MIMO IBC $\rightarrow$ SISO IC (in our context):
  - $h_{k\ell}$:= channel $\mathrm{Tx}(k) \rightarrow \mathrm{Rx}(\ell)$, $\sim \mathcal{CN}(0,1)$, *scaling*: $|h_{k\ell}|^2 = G_{k\ell}$, $\forall \ell, k \in \mathcal{L}$
  - *Alternatively*: $h_{k\ell} \in \mathbb{R}$, equal to $\sqrt{G_{\ell k}}$, $\forall k, \ell \in \mathcal{L}$

Weighted MMSE approach to dif. backlog WSRM

Weighted sum-MSE Minimization

$$\min_{\{w_\ell, u_\ell, v_\ell\}_{\ell\in\mathcal{L}}} \sum_{\ell=1}^{L} D_\ell(t)\left(w_\ell e_\ell - \log(w_\ell)\right)$$

$$\text{s. t.} \quad |v_\ell|^2 \leq P_\ell\ , \quad \ell\in\mathcal{L}$$

- $w_\ell^{opt} = e_\ell^{-1}, \quad \forall \ell \in \mathcal{L}$
- $u_\ell^{opt} \equiv u_\ell^{mmse} = \frac{h_{\ell\ell} v_\ell}{\sum_{k=1}^{L} |h_{k\ell}|^2 |v_k|^2 + V_\ell}, \quad \forall \ell\in\mathcal{L}$

- e_ℓ := m.sq. error, $w_\ell > 0$:= weight var, $v_\ell / u_\ell \in \mathbb{C}^{1\times 1}$:= gain Tx(ℓ)/Rx(ℓ)

Simplifying WMSE minimization yields BPPC at physical layer

$$\max_{\{v_\ell\}_{\ell\in\mathcal{L}}} \sum_{\ell=1}^{L} D_\ell(t) \log\left(1 + \frac{|h_{\ell\ell}|^2|v_\ell|^2}{\sum_{\substack{k=1\\k\neq\ell}}^{L} |h_{k\ell}|^2 |v_k|^2 + V_\ell}\right)$$

$$\text{s. t.} \quad |v_\ell|^2 \leq P_\ell\ , \quad \ell\in\mathcal{L}$$

Upon change of variables:

- $p_\ell = |v_\ell|^2, \forall \ell\in\mathcal{L}$
- $G_{k\ell} = |h_{k\ell}|^2, \forall k,\ell\in\mathcal{L}$

Iterative WMMSE algorithm for BPPC

Batch iterative WMMSE algorithm for BPPC

1. $\forall t \geq 1$: $\rightarrow D_\ell(t), \forall \ell \in \mathcal{L}$
2. *Initialize*: $v_\ell = \sqrt{P_\ell}, \forall \ell \in \mathcal{L}$
3. **repeat**
4. $(w_\ell)' \leftarrow w_\ell, \forall \ell \in \mathcal{L}$
5. $u_\ell \leftarrow \frac{h_{\ell\ell} v_\ell}{\sum_{k=1}^{L} G_{k\ell} v_k^2 + V_\ell}, \forall \ell \in \mathcal{L}$
6. $w_\ell \leftarrow (1 - u_\ell h_{\ell\ell} v_\ell)^{-1}, \forall \ell \in \mathcal{L}$
7. $v_\ell \leftarrow \left[\frac{D_\ell(t) w_\ell u_\ell h_{\ell\ell}}{\sum_{k=1}^{L} D_k(t) w_k u_k^2 G_{\ell k}}\right]_0^{\sqrt{P_\ell}}, \forall \ell \in \mathcal{L}$
8. **until** $|\log(w_\ell) - \log((w_\ell)')| \leq \epsilon, \forall \ell \in \mathcal{L}$
9. set $p_\ell(t) := v_\ell^2, \forall \ell \in \mathcal{L}$

- $(\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w})$ – space
- **convex** w.r.t. to ***each*** $\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}$ holding others ***fixed***
- block coordinate descent technique $\rightarrow$ ***suboptimal*** solution
- Feedback: $\left\{D_k u_k^2 w_k\right\}_{k \neq \ell} \rightarrow \ell$
- ***Consensus-on-the-min*** algorithm for termination of iterates
- **one shot approximation** to BPPC $\forall t$
- **low** complexity

Distributed Adaptive WMMSE algorithm for BPPC

- Same stable setup considered; exploit expected periodicity due to push-pull nature of solution
- Same strategy for warm re-start with the S.A. algorithm
- speed up convergence of WMMSE

Distributed Adaptive WMMSE for BPPC (warm re-start)

- ***Power - initialization***:
  For $t = 1$ set: $v_\ell = \sqrt{P_\ell}$, $\forall \ell \in \mathcal{L}$
  For $t \in [2, W]$ set:

$$\tau_o = \arg \min_{\tau \in \{t-1,\ldots,1\}} |D_\ell(t) - D_\ell(\tau)|, \ \ v_\ell = \sqrt{p_\ell(t-\tau_o)}, \forall \ell \in \mathcal{L}$$

  For $t \geq W+1$ set:

$$\tau_o = \arg \min_{\tau \in \{t-1,\ldots,t-W\}} |D_\ell(t) - D_\ell(\tau)|, \ \ v_\ell = \sqrt{p_\ell(t-\tau_o)}, \forall \ell \in \mathcal{L}$$

- $W >$ expected period

Scheduling heuristic

- Consider $\mathcal{A} \neq \mathcal{B}$ subsets of links: $\mathcal{A} \dashrightarrow i$ and $i \dashrightarrow \mathcal{B}$
- BPPC (under our specific setups / assumptions): node i ***favors*** subset with **maximum** sum of differential backlogs
- Each node $i \in \{\mathcal{N}\} \setminus N$ knows backlogs of all other nodes $j \neq i, N$

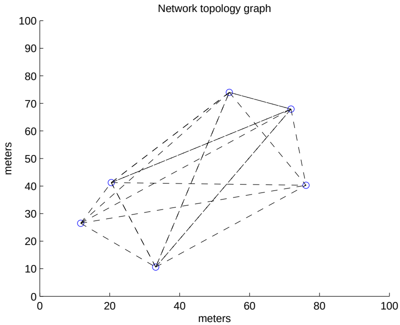
- $\ell = (i, j),\ j \neq N$
  $D_\ell(t)$: node j, $W_j \to i$
- $\forall$ links departing from i
- $\forall$ nodes except N (sink)
- possible only in our specific setups
- $\downarrow \sharp$ of variables $\to$
- $\downarrow$ computational complexity
- mode 2 implementation

Scheduling heuristic (network layer)

For each node $i \in \{\mathcal{N}\} \setminus N$:

- Calculate $D_\ell(t),\ \forall \ell : \mathrm{Tx}(\ell) = i$,
  $$D_\ell(t) := \begin{cases} \max\{0, W_i(t) - W_j(t)\}, & j \neq N \\ W_i(t), & j = N. \end{cases}$$
- Calculate $D_k(t),\ \forall k : \mathrm{Rx}(k) = i$,
  $D_k(t) := \max\{0, W_j(t) - W_i(t)\}, j \neq N.$
- If $\sum_{\ell:\mathrm{Tx}(\ell)=i} D_\ell(t) > \sum_{k:\mathrm{Rx}(k)=i} D_k(t)$,
  set $D_k(t) = 0, \forall k : \mathrm{Rx}(k) = i$,
  else
  set $D_\ell(t) = 0,\ \forall \ell : \mathrm{Tx}(\ell) = i$

Simulation setup



- $N = 6$ nodes, low-left = s, top-right = d, $L = 25$ links
- $G_{\ell,k} \sim 1/d^4$, d : distance Tx(ℓ), Rx(k), $G = 128$,
- no-listen-while-talk: if $\text{Rx}(\ell) = \text{Tx}(k) \rightarrow G_{k,\ell} = 1/\text{eps}$
- $V_\ell = 10^{-12}$, $P_\ell = 5$, $\forall \ell$
- Deterministic fixed-rate arrivals

Performance evaluation of core step algorithms

Simulation experiment:

- 100 packets/control slots, deterministic arrival rate: 9 packets/slot
- Centralized *Batch high-SINR*$\rightarrow$ resulting $D_\ell(t)$, $\forall \ell$, $\forall t \in \{1, 100\}$
- *High-SINR* $\leftrightarrow$ $\alpha_\ell(t) = 1$, $\beta_\ell(t) = 0$, $\forall \ell$ and $\forall t$
- Solve above 100 instances via distributed *core step* algorithms:
  - *Initialization* : $\{\lambda_\ell\}_{\ell \in \mathcal{L}}$, $\{\gamma_{\ell k}\}_{k \neq \ell}^{\ell \in \mathcal{L}} \rightarrow 1$, $\left\{\tilde{\boldsymbol{i}}_\ell\right\}_{\ell \in \mathcal{L}}$ random, $\delta_s = \delta = 0.01$, $\forall s$
  - *Termination criterion*: local metrics must drop under $\epsilon = 10^{-2}$, $\forall \ell \in \mathcal{L}$
  - variation of local augmented Lagrange function, $\forall \ell \in \mathcal{L}$
  - norm of residual consensus constraints vector $\mathbf{r}_\ell(s)$, $\forall \ell \in \mathcal{L}$

The role of the penalty parameter

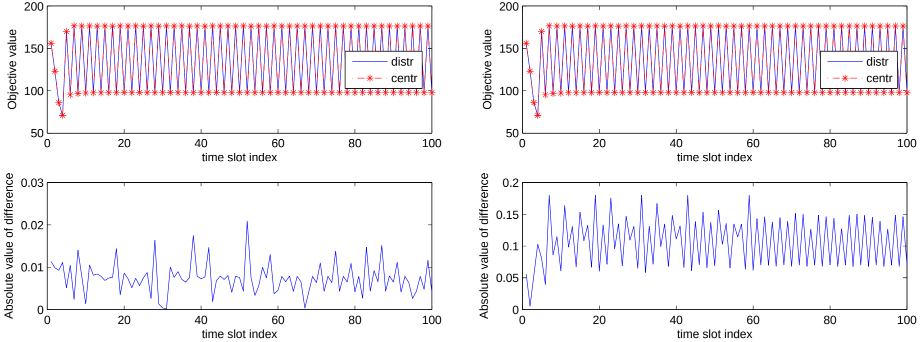
Table: Results for problem instance at 30^{th} slot, for various ρ and initial $\{\rho_{\ell,o}\}_{\ell \in \mathcal{L}}$. Tolerance $\epsilon := 10^{-2}$. Objective value for Batch high-SINR: 97.6388.

ρ / $\{\rho_{\ell,o}\}_{\ell \in \mathcal{L}}$	0.002	0.003	0.01	0.1	0.5	1	5
core step 1: $\sharp$ iterations	1058	732	318	106	311	463	1212
core step 1: objective value	97.71	97.7	97.67	97.52	97.49	97.42	97.07
core step 2: $\sharp$ iterations	127	103	103	97	125	127	256
core step 2: objective value	97.58	97.56	97.58	97.57	97.57	97.57	97.65

- ‘sweet spot’ at $\rho \sim 0.1$: **minimum** $\sharp$ of iterations required
- Very low or high values of $\rho \rightarrow$ *slow down convergence.*
- Adaptive $\{\rho_\ell\}_{\ell \in \mathcal{L}}$: core step 2 less dependent on the initial choice

Simulation results

- Distributed core step algorithms vs centralized high-SINR, for 100 problem instances



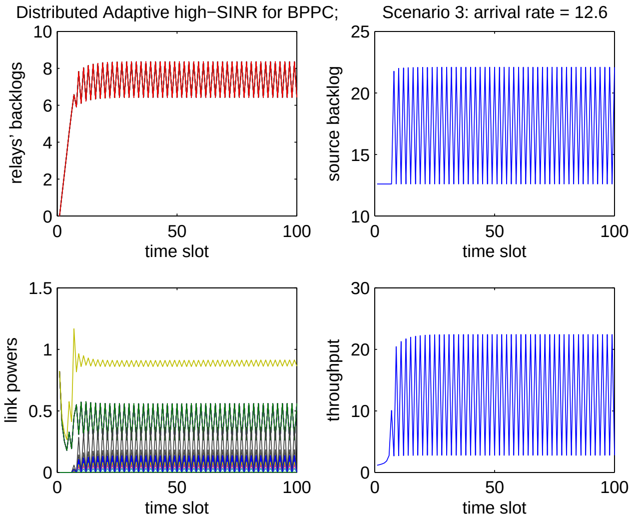
Core step 1 (left): Objective value (top), abs. difference (bottom), $\rho = 0.01$.
Core step 2 (right): Objective value (top), abs. difference (bottom), $\rho = 0.1$.

Performance evaluation of distributed S.A. algorithms for BPPC

- Performance of distributed high-SINR / S.A. examined through simulations
  - Scenario 1: ***Small network*** ($N = 6$, $L = 25$) nodes, ***moderate interference*** ($G = 128$)
  - Scenario 2: ***Small network*** ($N = 6$, $L = 25$) nodes, ***stronger interference*** ($G = 8$)
  - Scenario 3: ***Larger network*** ($N = 12$, $L = 121$) nodes, ***moderate interference*** ($G = 128$)
- Fixed physical layer propagation conditions
- Deterministic fixed-rate arrivals
- no-listen-while-talk: if $\mathrm{Rx}(\ell) = \mathrm{Tx}(k) \rightarrow G_{k,\ell} = 1/\mathrm{eps}$
- $V_\ell = 10^{-12}$, $P_\ell = 5$, $\forall \ell$

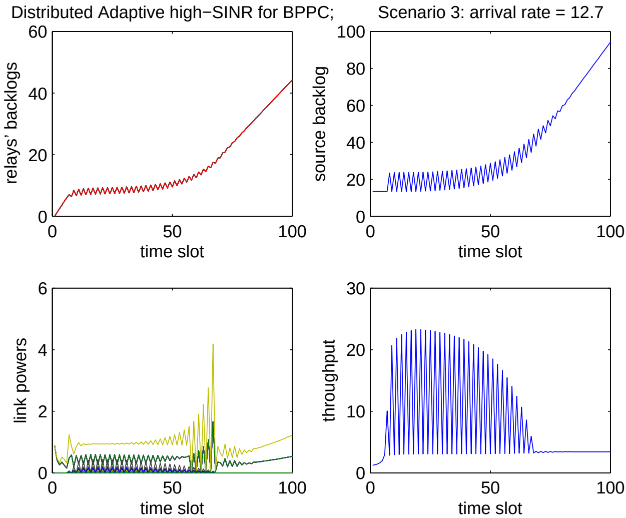
Distributed adaptive high-SINR

- Scenario 3- larger network; $N = 12$, $G = 128$, max stable rate: 12.6 pps



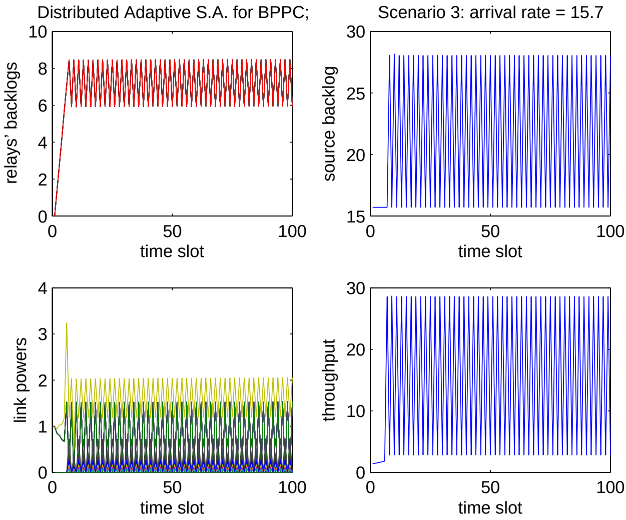
Distributed adaptive high-SINR

- Scenario 3- unstable setup; arrival rate: 12.7 pps



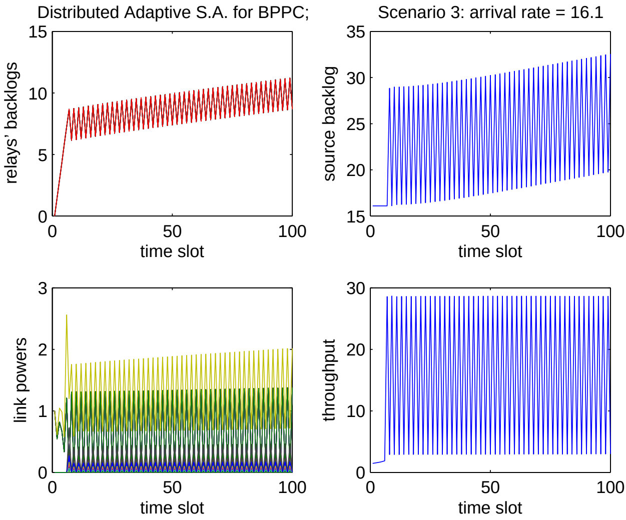
Distributed adaptive Successive Approximation

- Scenario 3- larger network; $N = 12$, $G = 8$, max stable rate: 15.7 pps



Distributed adaptive Successive Approximation

- Scenario 3- unstable setup; arrival rate: 16.1 pps



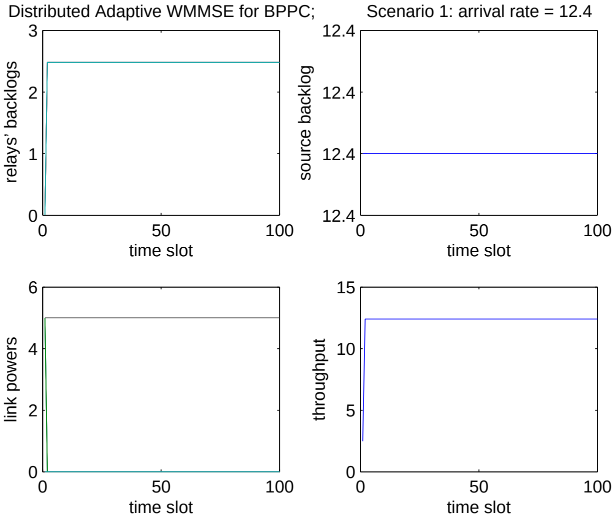
Summarizing simulation results

Table: Maximum stable throughput and average weighted sum-rate attained by all algorithms in all scenarios. (1):= core step 1, (2):= core step 2, w.s.r. := weighted sum-rate.

Scen. 1	Batch high-SINR (1) / (2)	Adapt. high-SINR (1) / (2)	Batch S.A. (1) / (2)	Adapt. S.A. (1) / (2)
rate	9.7 / 9.7	9.7 / 9.7	10.4	10.4
w.s.r	151.32 / 151.32	151.31/ 151.31	184.46	184.45
Scen. 2				
rate	2.4 / 2.4	2.4 / 2.4	7.8	7.8
w.s.r.	0.95 / 0.96	0.95 / 0.96	107.48	107.47
Scen. 3				
rate	12.6 / 12.6	12.6 / 12.6	15.7	15.7
w.s.r.	276.19 / 276.21	276.18 / 276.2	454.31	454.3

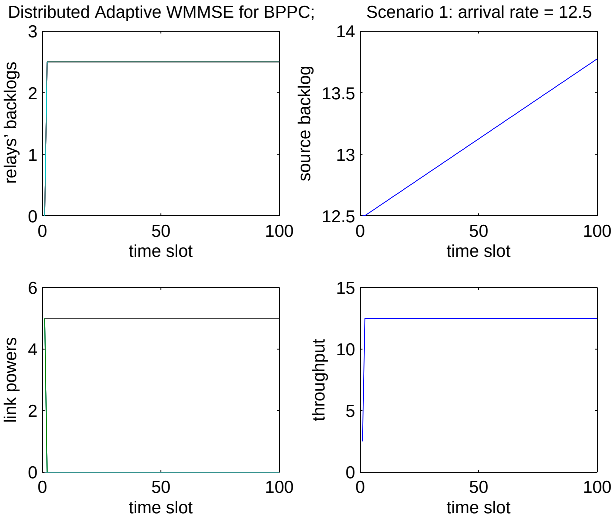
Adaptive WMMSE

- Scenario 1- maximum stable rate: 12.4 pps > S.A.-BPPC (10.4 pps)



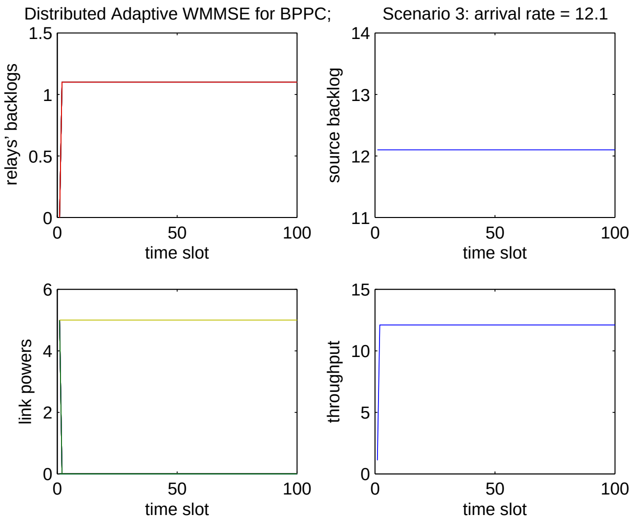
Adaptive WMMSE

- Scenario 1- unstable setup; arrival rate: 12.5 pps



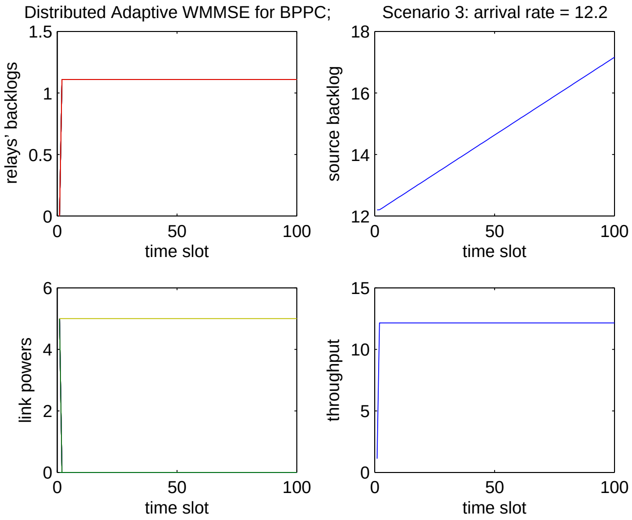
Adaptive WMMSE

- Scenario 3- larger network; max stable rate: 12.1 pps < S.A. (15.7 pps)



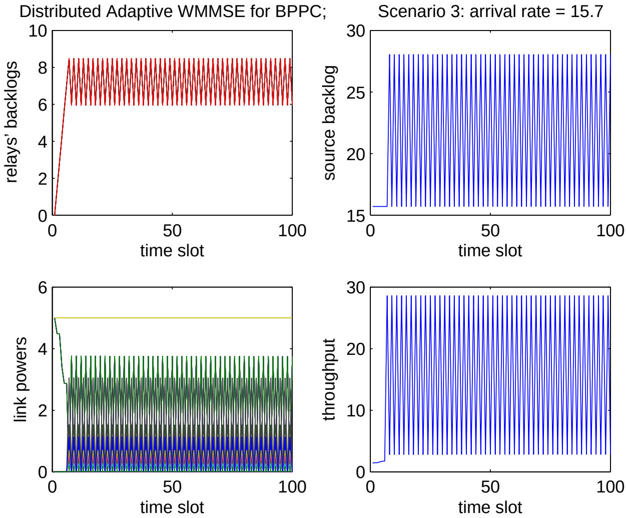
Distributed adaptive high-SINR

- Scenario 3- unstable setup; arrival rate: 12.2 pps



Adaptive WMMSE (mode 2)

- Scenario 3 - maximum stable rate: 15.7 pps (same for S.A.)

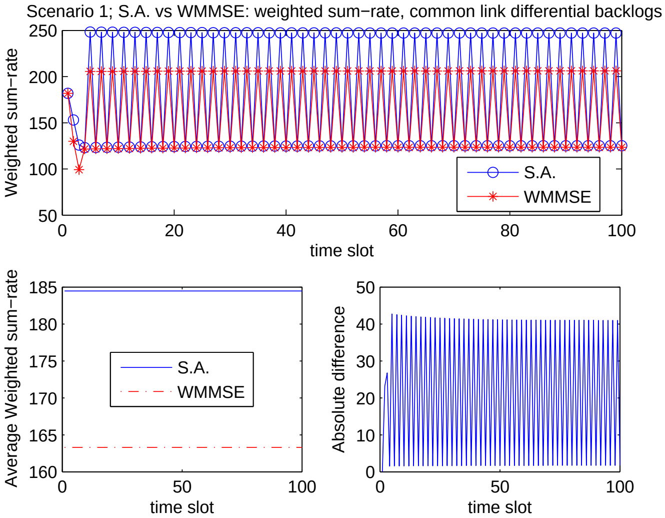


WMMSE versus S.A. under identical problems

- Fair comparison w.r.t. weighted sum-rate
- $\rightarrow$ under identical problem instances
- Experiment:
- Let batch S.A. drive the network (schedule all links with $D_\ell(t) > 0, \forall \ell, \forall t$) (mode 1)
- Solve problem with dif. backlogs resulting at each time slot from S.A. via batch WMMSE
- Test WMMSE under mode 1, mode 2

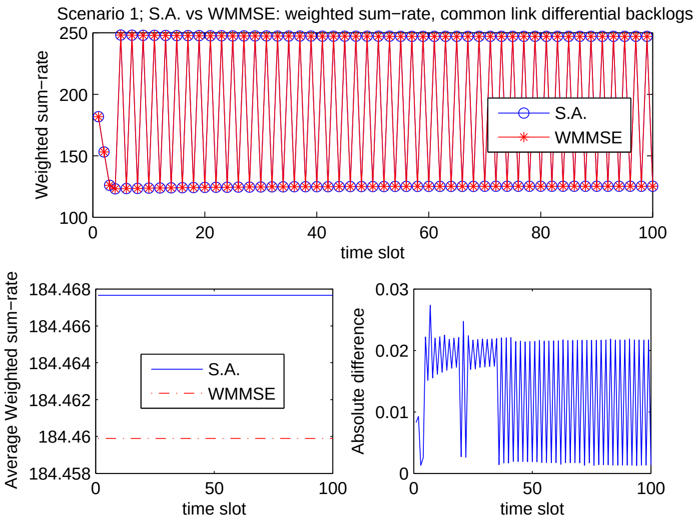
WMMSE versus S.A. in scenario 1

- Rate: 10.4 pps, w.s.r. batch S.A. vs w.s.r. batch WMMSE (mode 1)



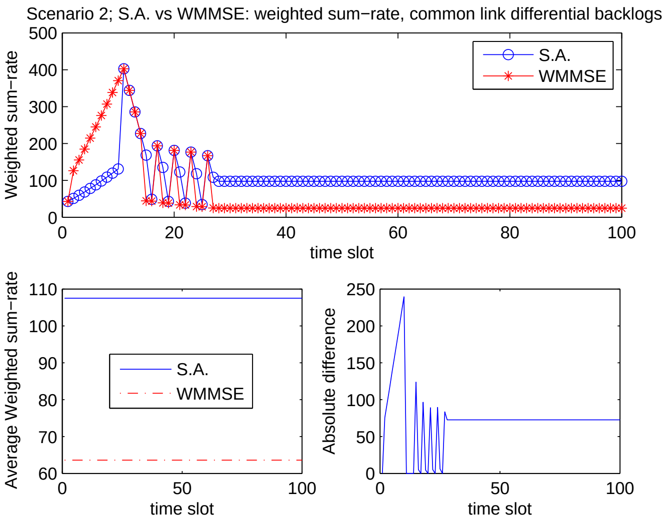
WMMSE versus S.A. in scenario 1

- Rate: 10.4 pps, w.s.r. batch S.A. vs w.s.r. batch WMMSE (mode 2)



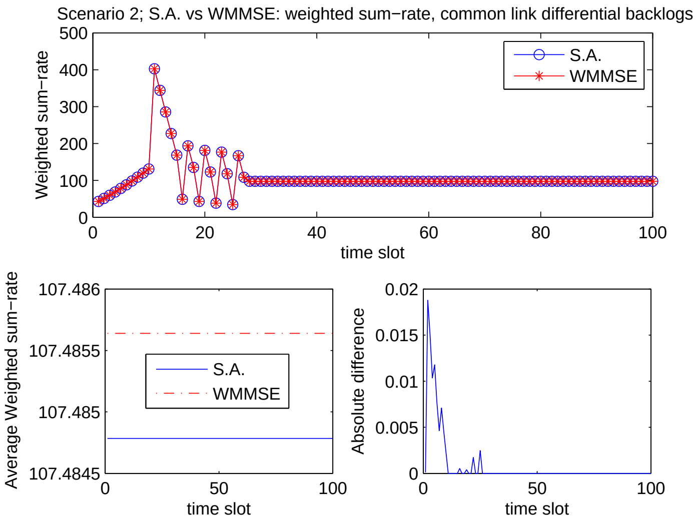
WMMSE versus S.A. in scenario 2

- Rate: 7.8 pps, w.s.r. batch S.A. vs w.s.r. batch WMMSE (mode 1)



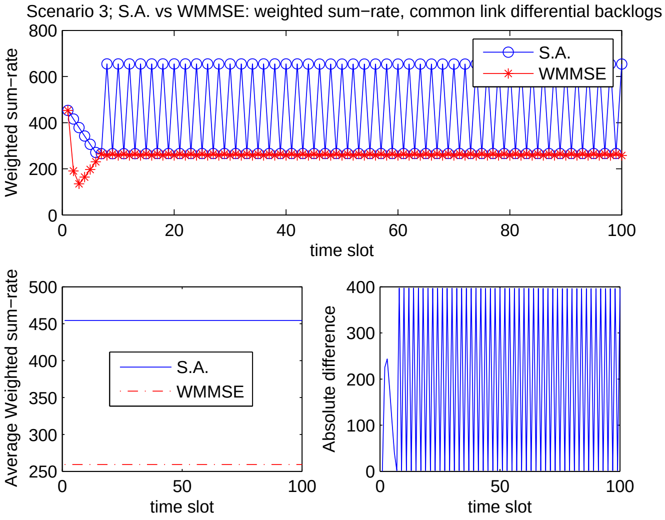
WMMSE versus S.A. in scenario 2

- Rate: 7.8 pps, w.s.r. batch S.A. vs w.s.r. batch WMMSE (mode 2)



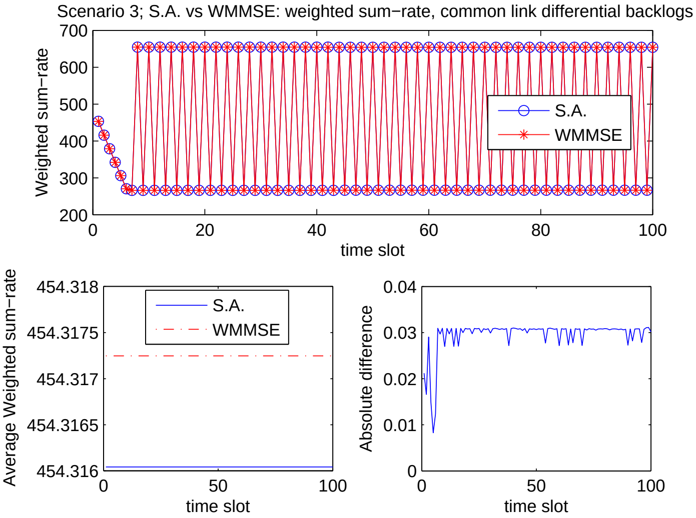
WMMSE versus S.A. in scenario 3

- Rate: 15.7 pps, w.s.r. batch S.A. vs w.s.r. batch WMMSE (mode 1)



WMMSE versus S.A. in scenario 3

- Rate: 15.7 pps, w.s.r. batch S.A. vs w.s.r. batch WMMSE (mode 2)



Summarizing simulation results

Table: Average throughput and average weighted sum-rate attained by WMMSE and S.A. algorithms in all scenarios. w.s.r. := weighted sum-rate.

Scen. 1	WMMSE (Batch / Ad.) mode 1	WMMSE (Batch / Ad.) mode 2	S.A. (Batch / Ad.) mode 1	S.A. (Batch / Ad.) mode 2
Max rate	12.4	10.4	10.4	10.4
Avg. w.s.r	155.4	184.4	184.4	184.4
Scen. 2				
Max rate	12.3	7.8	7.8	7.8
Avg. w.s.r.	168.6	107.3	107.4	107.4
Scen. 3				
Max rate	12.1	15.7	15.7	15.7
Avg. w.s.r.	149	454.4	454.3	454.3

Average complexity of WMMSE

Table: Best and worst cases of average run-time and average number of iterations per slot, of all WMMSE-based algorithms, concerning all scenarios considered.

	Batch WMMSE mode 1	Adaptive WMMSE mode 1	Batch WMMSE mode 2	Adaptive WMMSE mode 2
Max avg. run-time	0.06	0.0125	0.114	0.008
Min. avg. run-time	0.015	0.0061	0.055	0.0045
Max avg. $\sharp$ iter.	118	1	376	3
Min avg. $\sharp$ iter.	20	1	242	1

- Batch S.A. is not comparable (orders of magnitude slower)
- Adaptive S.A.: min avg. run-time 0.0065 seconds, max: 0.2 seconds (steady state)

Concluding remarks

- WMMSE clearly better than S.A.-BPPC complexity-wise
- Inconclusive results regarding performance:
  - WMMSE outperforms in strong interference scenario
  - S.A.-BPPC holds the edge in larger networks
- Comparing ***suboptimal*** BPPC solutions w.r.t. average weighted sum-rate may be a ***fallacy*** → which algorithm supports higher stable throughput?
- Scheduling heuristic ***improves throughput performance*** of WMMSE in larger networks
- Improvement to original WMMSE is simple but worthwhile, effective power initialization

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- Parts of this work appear in:

- E. Matskani, N.D. Sidiropoulos, and L. Tassiulas, "Convex Approximation Algorithms for Back-pressure Power Control," *IEEE Transactions on Signal Processing*, vol. 60, no. 4, pp. 1957-1970, April 2012
- E. Matskani, N.D. Sidiropoulos, and L. Tassiulas, "Convex Approximation Algorithms for Back-pressure Power Control of Wireless Multi-hop Networks," in *Proc. IEEE ICASSP 2011*, pp. 3032-3035, Prague, Czech Republic, May 22-27, 2011
- E. Matskani, N.D. Sidiropoulos, and L. Tassiulas, "Distributed Back-pressure Power Control for Wireless Multi-hop Networks," in *Proc. IEEE ICASSP 2012*, pp. 3032-3035, Kyoto, Japan, May 25-30, 2012