Technical University of Crete

School of Mineral Resources Engineering
Geostatistics Research Unit

School of Environmental Engineering
Lab of Geoenvironmetal Engineering

"Bayesian Approach for Estimating the Space-time Variability of Groundwater Level in a Sparsely Monitored Basin on a Mediterranean Island"

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European Union
European Social Fund

OPERATIONAL PROGRAMME
EDUCATION AND LIFELONG LEARNING
investing in knowledge society
MINISTRY OF EDUCATION, LIFELONG LEARNING AND RELIGIOUS AFFAIRS
MANAGING AUTHORITY

NSRF 2007-2013
programme for development
EUROPEAN SOCIAL FUND

Co- financed by Greece and the European Union

Aim and Scope

- Combination of spatial and temporal groundwater level measurements to exploit **spatiotemporal correlations for** monitoring and prediction

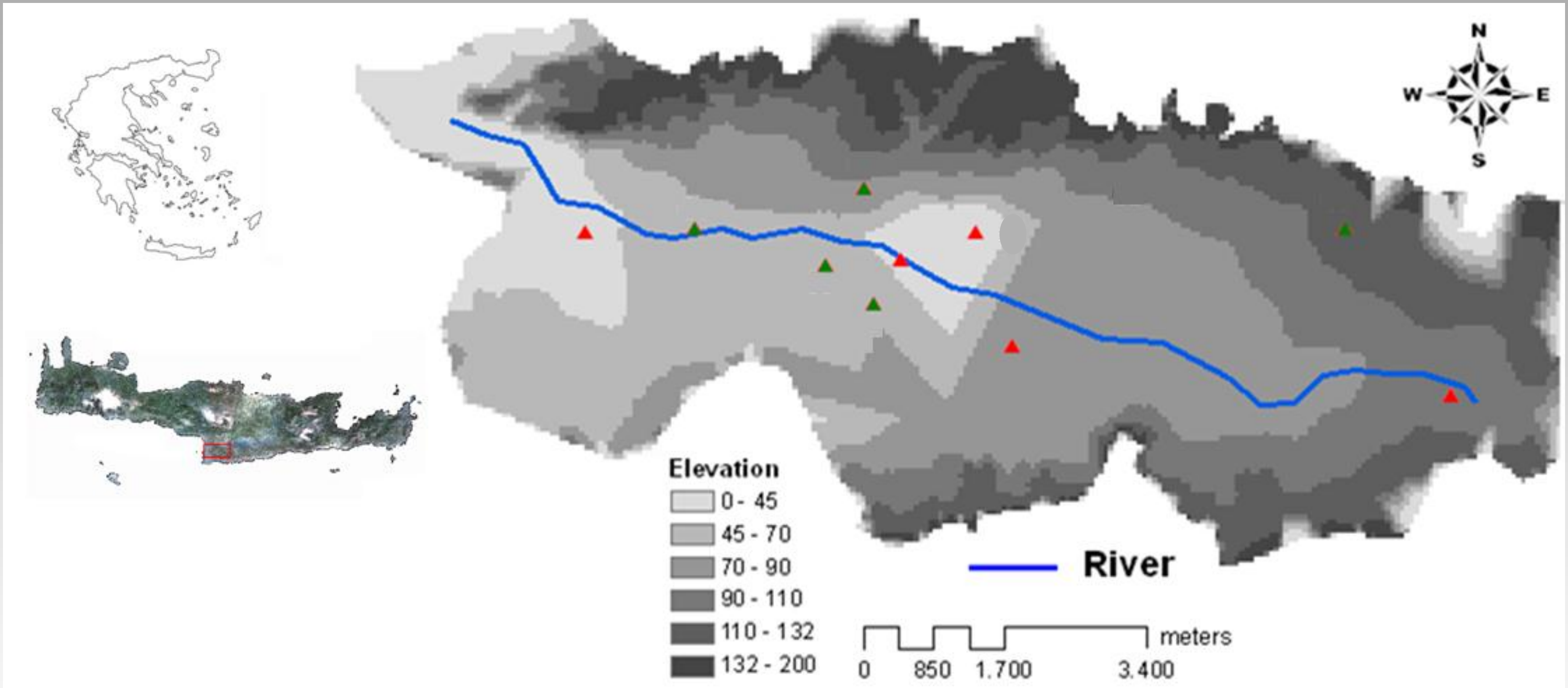
- Introduction of a **non-separable Spartan spatiotemporal variogram** model

- **Geostatistical methods** in association with **auxiliary information** in groundwater level applications provide:
  - Improved space-time visualization of the aquifer free surface
  - Maximization of the information regarding groundwater level space-time variability

- **Performance evaluation** of *spatial model* using the spatiotemporal residual kriging (**STRK**) interpolation methodology

- **Spartan** spatiotemporal variogram **prediction efficiency** is compared with
  - Product-sum (P-S) covariance function
  - Non-separable function based on the diffusion equation (ST-D)

Study Area: Mires Basin, Messara Valley, Crete



10 wells are monitored between the years 1981 and 2009 bi-annually

Spatiotemporal Model

Spatiotemporal trend-fluctuation model: $H(\mathbf{s},t) = m_H(\mathbf{s},t) + h_f(\mathbf{s},t)$

Spatiotemporal trend: $m_H(\mathbf{s},t) = \hat{m}_H(t) \times m_H(\mathbf{s})$

Spatial Trend: Thiem's equation for unconfined aquifers:

$$\left(m_H(\mathbf{s})\right)^2 = H_0{}^2(\mathbf{s}) + \frac{1}{\pi K}\sum_{i=1}^{n} Q_i \ln\left(\frac{r_i}{R_i}\right), \quad r_i < R_i, \quad i = 1\ldots,n$$

- $H_0(\mathbf{s})$ is the initial groundwater level before pumping
- K is the hydraulic conductivity
- n is the number of wells (i=1…,n)
- $r_i = \|\mathbf{s} - \mathbf{s}_i\|$ is the distance of the estimation point from the *i*-th well
- Q_i is the pumping rate
- R_i is the well's radius of influence

(Varouchakis and Hristopulos 2013)

Temporal Trend: Exponentially-weighted moving average filter:

$$\hat{m}_H(t_i) = w\bar{H}(t_i) + (1-w)\hat{m}_H(t_{i-1})$$

$0 < w \leq 1$ is the weight of the temporal model

Spartan Covariance Function

Spartan covariance function in 3 *dimensions* *(Hristopulos and Elogne 2007)* :

$$C_H(\mathbf{h}) = \begin{cases} \dfrac{\eta_0 e^{-h\beta_2}}{2\pi\sqrt{|\eta_1^2-4|}}\left[\dfrac{\sin(h\beta_1)}{h\beta_1}\right], & \text{for } |\eta_1|<2,\ \sigma_z^2 = \dfrac{\eta_0}{2\pi\sqrt{|\eta_1^2-4|}} \\ \dfrac{\eta_0 e^{-h}}{8\pi}, & \text{for } \eta_1 = 2,\ \sigma_z^2 = \dfrac{\eta_0}{8\pi} \\ \dfrac{\eta_0(e^{-h\omega_1}-e^{-h\omega_2})}{4\pi(\omega_2-\omega_1)h\sqrt{|\eta_1^2-4|}}, & \text{for } \eta_1 > 2,\ \sigma_z^2 = \dfrac{\eta_0}{4\pi\sqrt{|\eta_1^2-4|}} \end{cases}$$

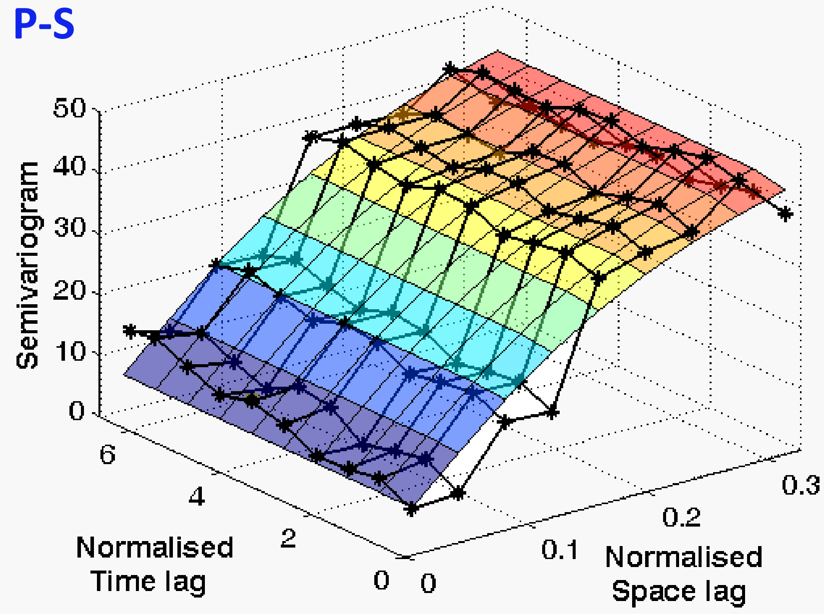
$$\|\mathbf{h}\| = \|\mathbf{r}\|/\xi$$

- η_0 is the scale factor, η_1 is the rigidity coefficient
- β_1 is a dimensionless wave number
- β_2 and $\omega_{1,2}$ are dimensionless damping coefficients
- ξ is a characteristic length

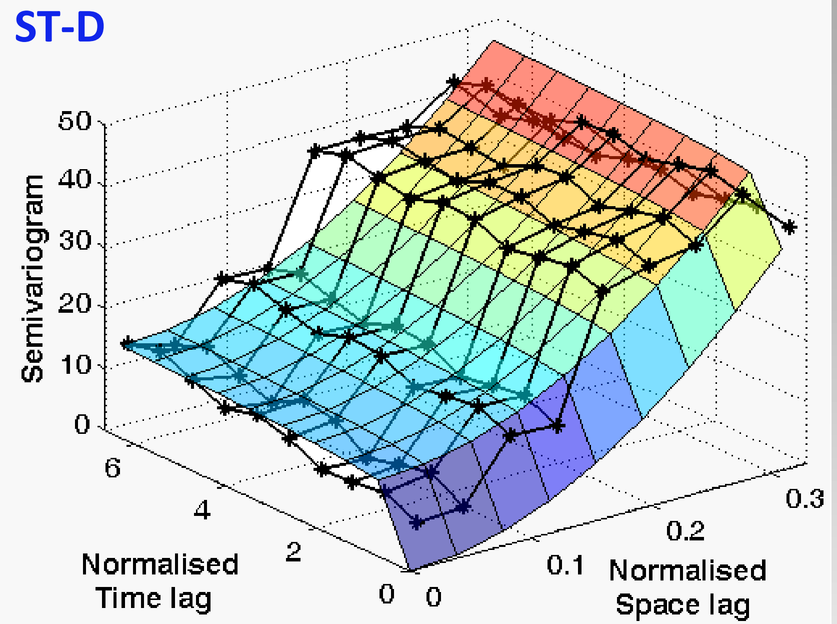
- $\boxed{\mathbf{h} = r/\xi}$ the normalized lag vector
- $h = |\mathbf{h}|$ its Euclidean norm
- σ_z^2 the variance
- α is ST scale factor

Space Time Variogram Models

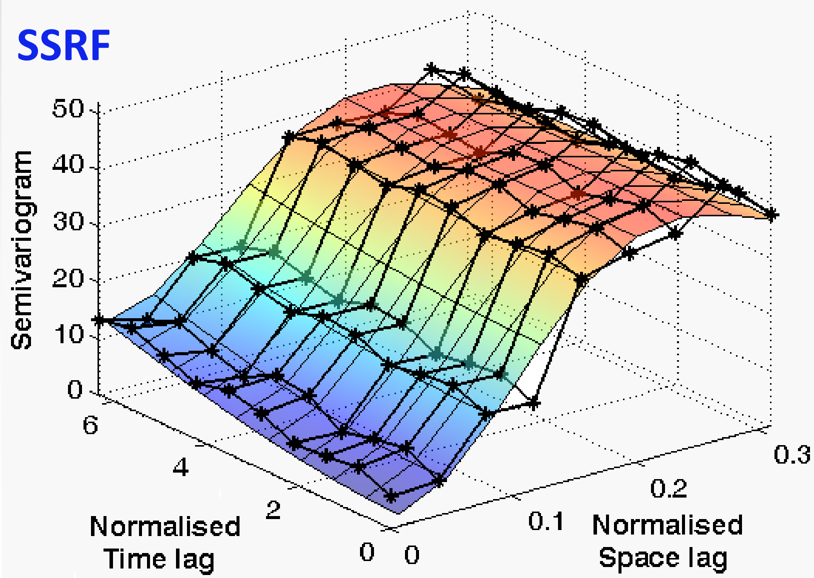
P-S



ST-D



SSRF



- **P-S:** Matérn spatial and an Exponential temporal component respectively

- **ST-D** is defined by the following function,

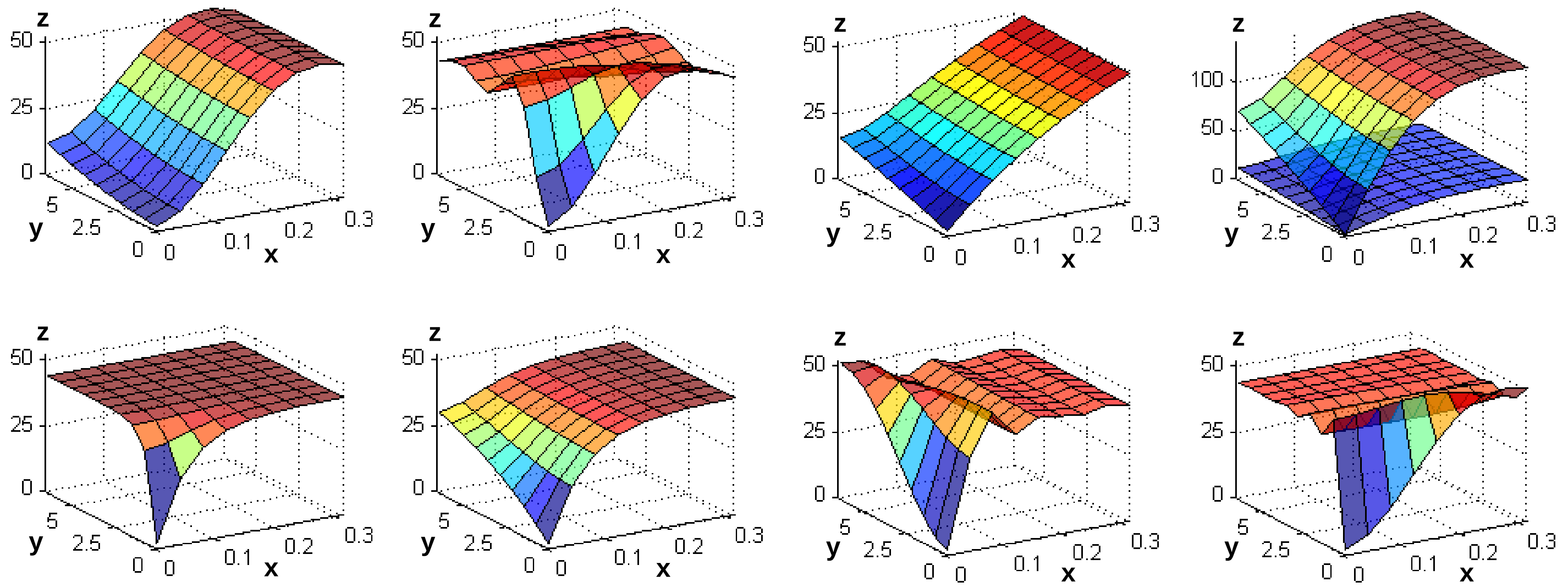
$$C_{ST}(\mathbf{r},\tau) \cong \left(\beta\tau^{2\gamma}+1\right)^{-n/2} \exp\left(-\mathbf{r}^2 / \left(\beta\tau^{2\gamma}+1\right)\right)$$
$$0 \le \beta \le 1,\ 0 < \gamma \le 1$$

where β, γ are the function's parameters and n are the spatial dimensions.

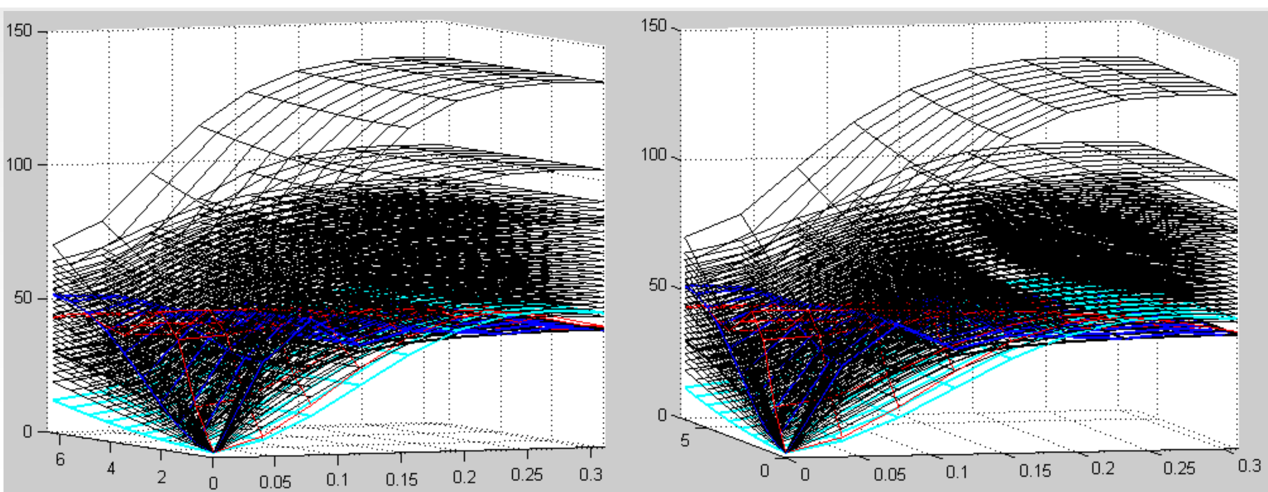
Cross Validation Analysis

Cross Validation Measures	2007-08 Wet period			2007-08 Dry period			2008-09 wet period	2008-09 dry period
	SSRF	ST-D	P-S	SSRF	ST-D	P-S	SSRF	SSRF
MARE	0.14	0.19	0.21	0.17	0.21	0.21	0.24	0.28
Modified coefficient of model efficiency, E’	0.90	0.82	0.82	0.89	0.83	0.79	0.79	0.75
Modified Index of Agreement, IoA’	0.91	0.86	0.87	0.89	0.80	0.80	0.79	0.73

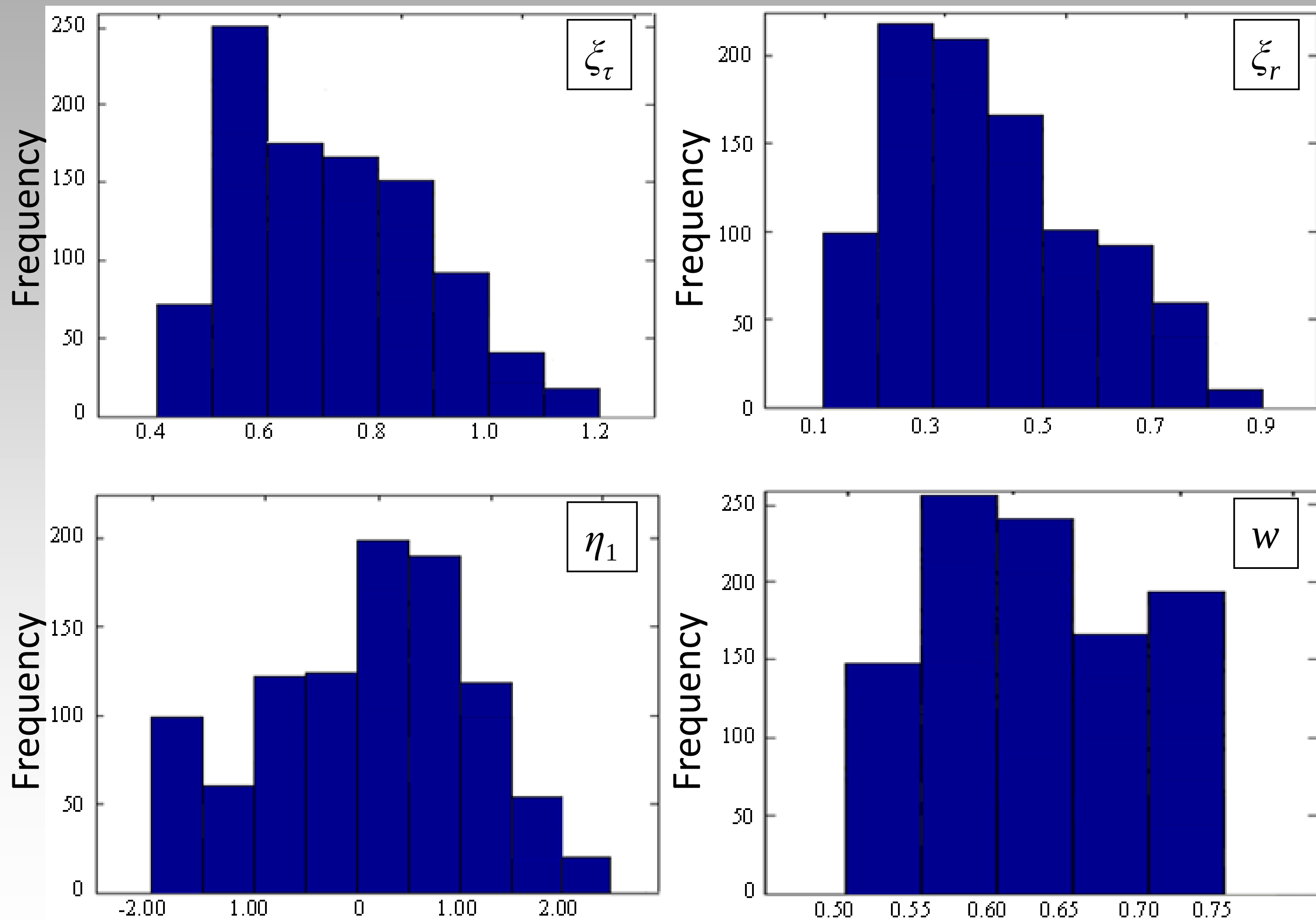
Representative Variograms obtained by Bootstrapping



- x axis: Normalized space lag
- y axis: Normalized time lag
- z axis: Variogram

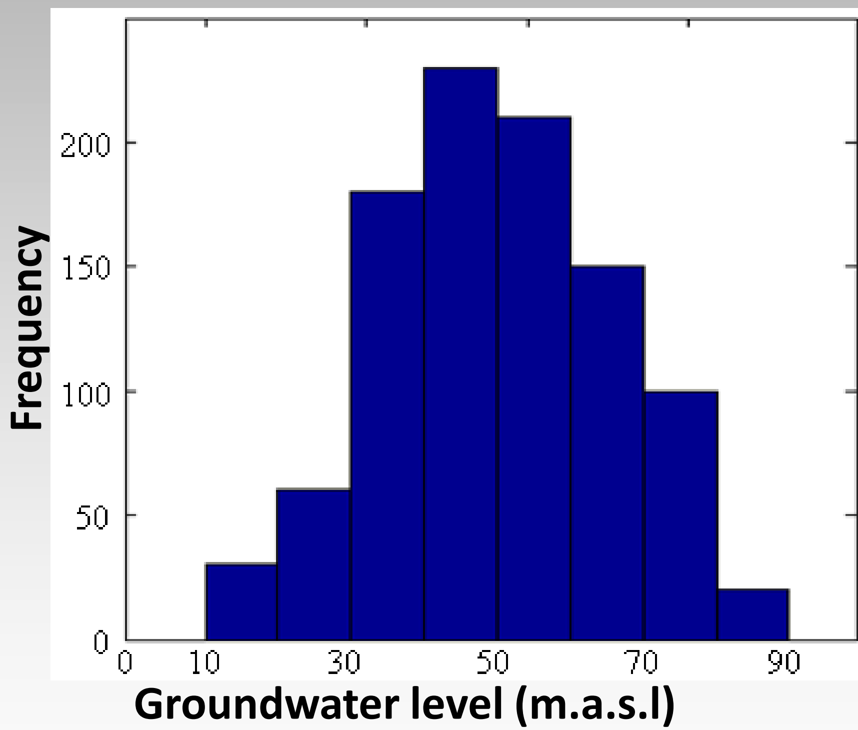


Sample of Posterior Parameter Distributions

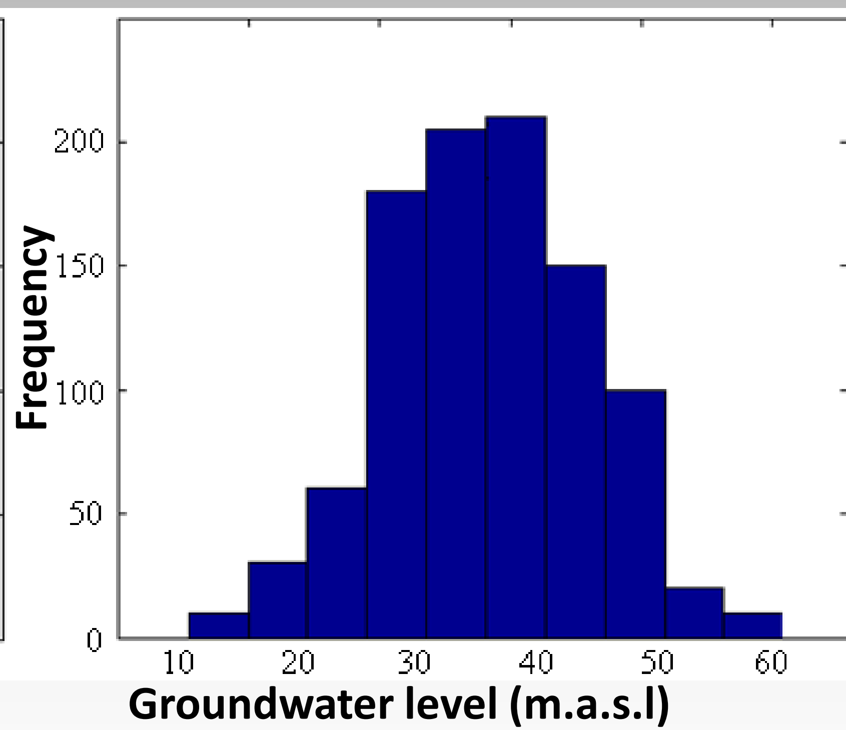


Predictive Probability Distribution at Grid Point

2007-08 Wet period

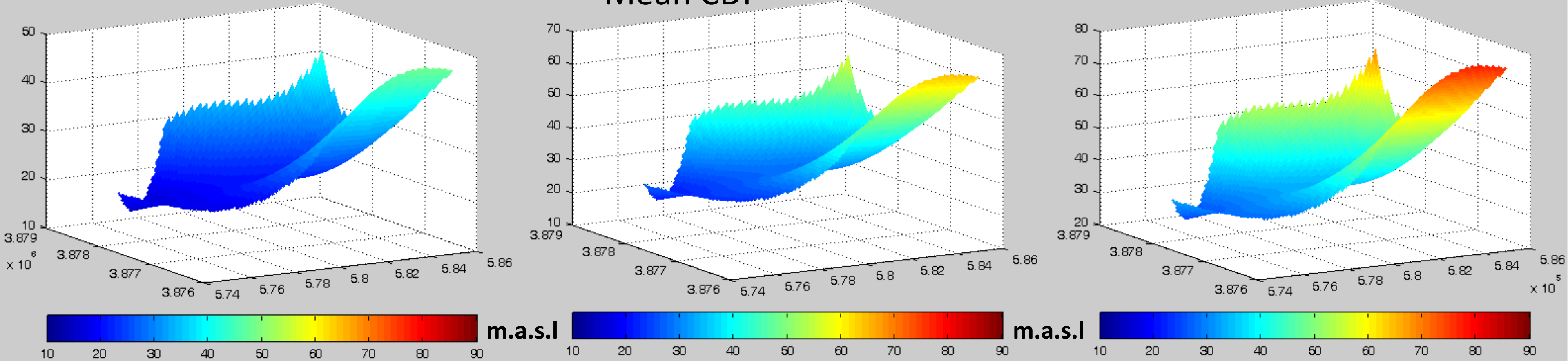


2007-08 Dry period



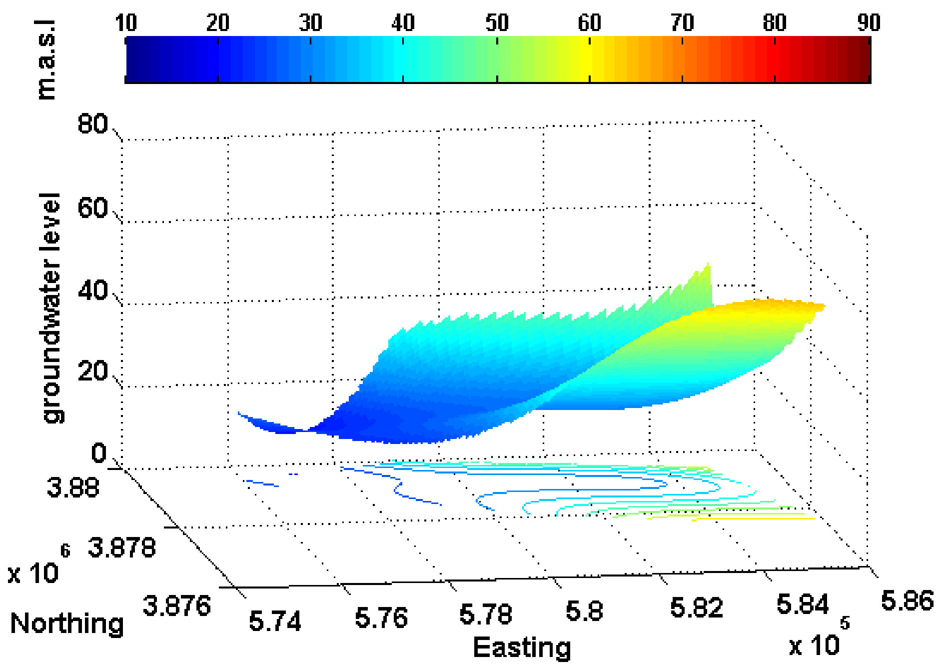
Uncertainty of Level Predictions: 2007-08 Wet Period

Mean CDF



5% CDF quantile

95% CDF quantile



Uncertainty of Level Predictions: 2007-08 Dry Period

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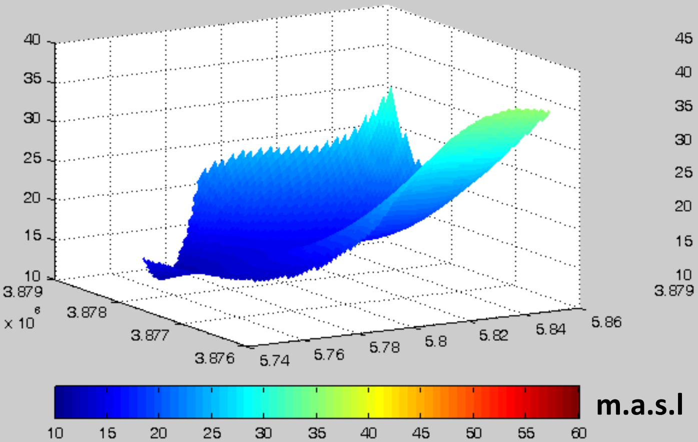

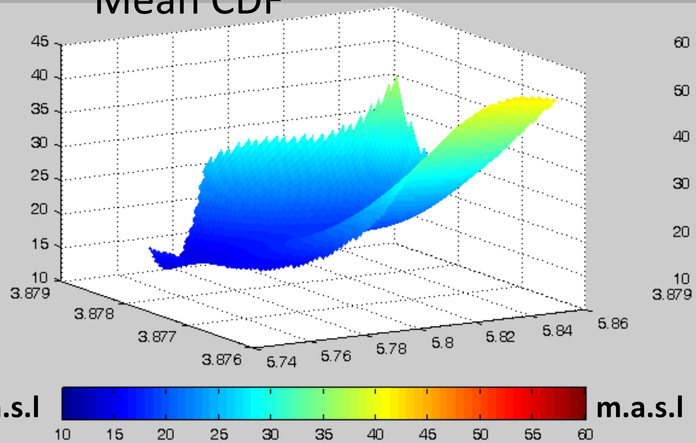

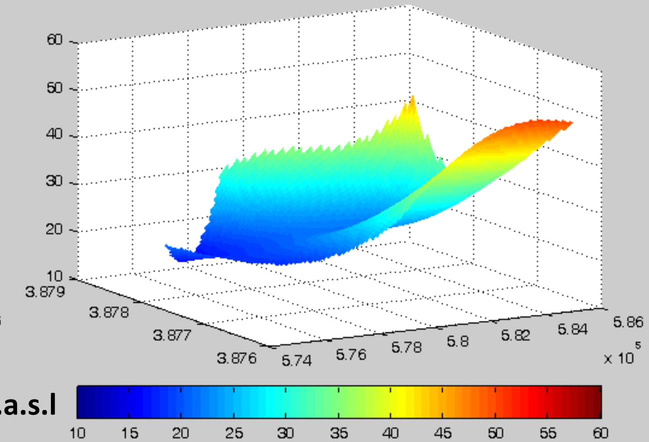

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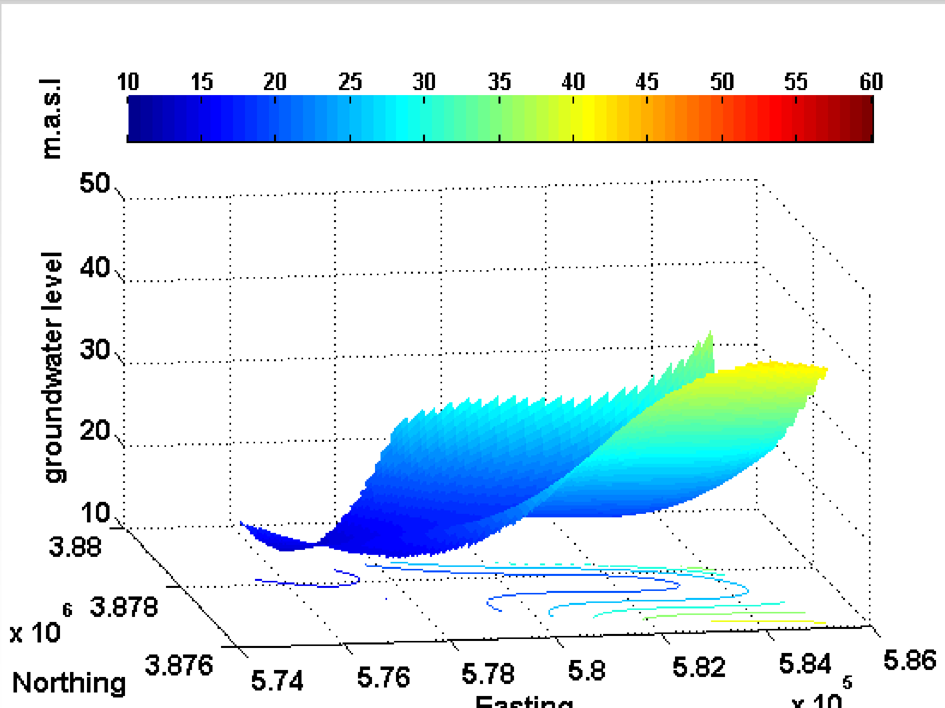

Conclusions

•The **Spartan** variogram model is a flexible new model for space-time correlations

•**STRK** using the **Spartan** variogram provided the **most accurate results**: 50% lower MARE than the P-S covariance function and 36% than the ST-D

•Incorporation of **auxiliary information** in the form physical laws (e.g., Thiem's law) in the trend improves the interpolation accuracy (22%)

•The **Bayesian Bootstrap** methodology provides a straightforward numerical framework for assessing the uncertainty of spatiotemporal model parameters and of the groundwater level predictions

Thank you very much for your attention

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